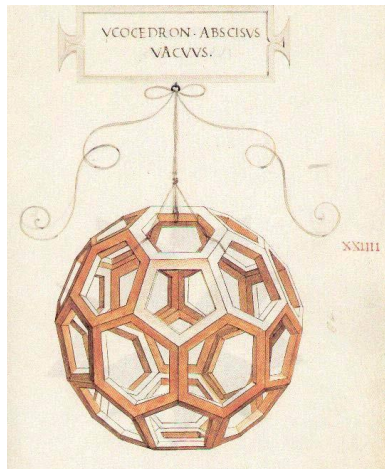


Fullerene-like architecture in nano-, micro- and macro-worlds



Eugene A. Katz

**Dept. of Solar Energy and Environmental Physics,
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Ben-Gurion University of the Negev,
Sede Boqer Campus**



Leonardo, 1509



A. Dürer, 1497-98

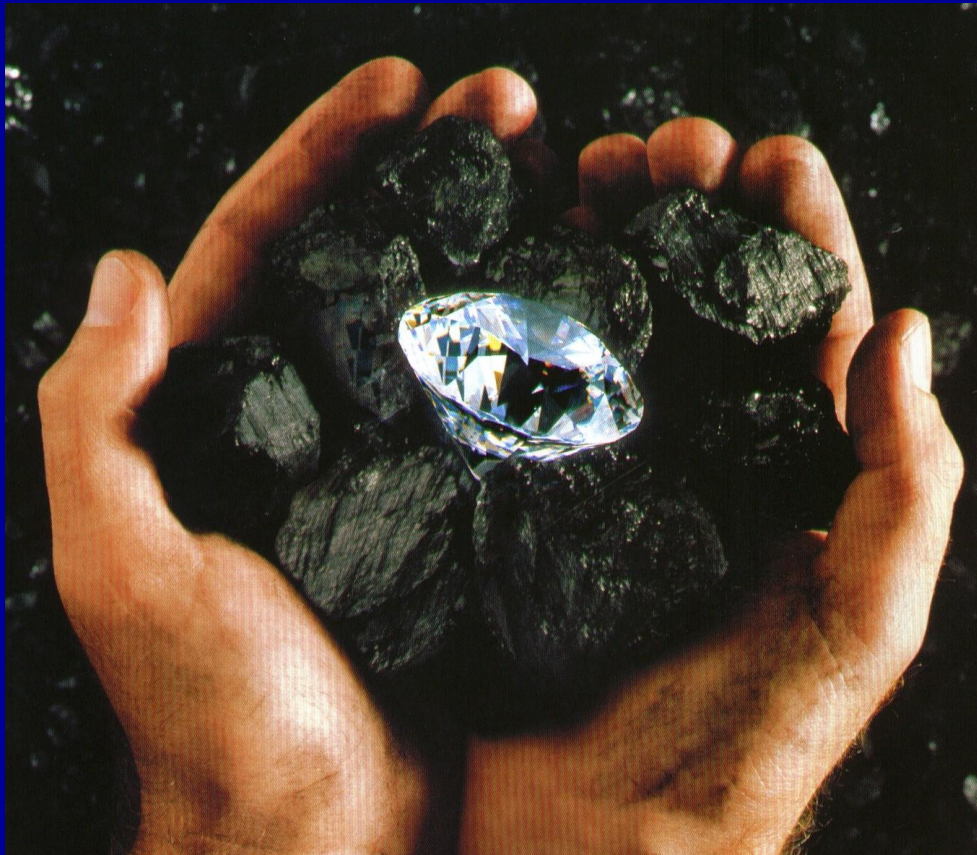


Contents

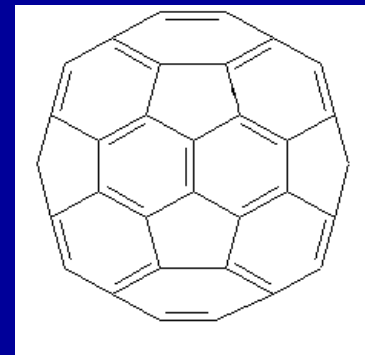
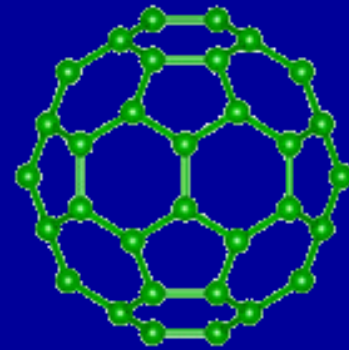
1. Discovery of C_{60} molecule
2. Buckminster Fuller's energetic geometry and geodesic domes
3. Long history of exploration of polyhedra: science and fine art
4. Leonhard Euler's theorem
5. Fullerene-like structure of molecules and nanoclusters, viruses, microorganisms and buildings: the same story

Allotropes of carbon

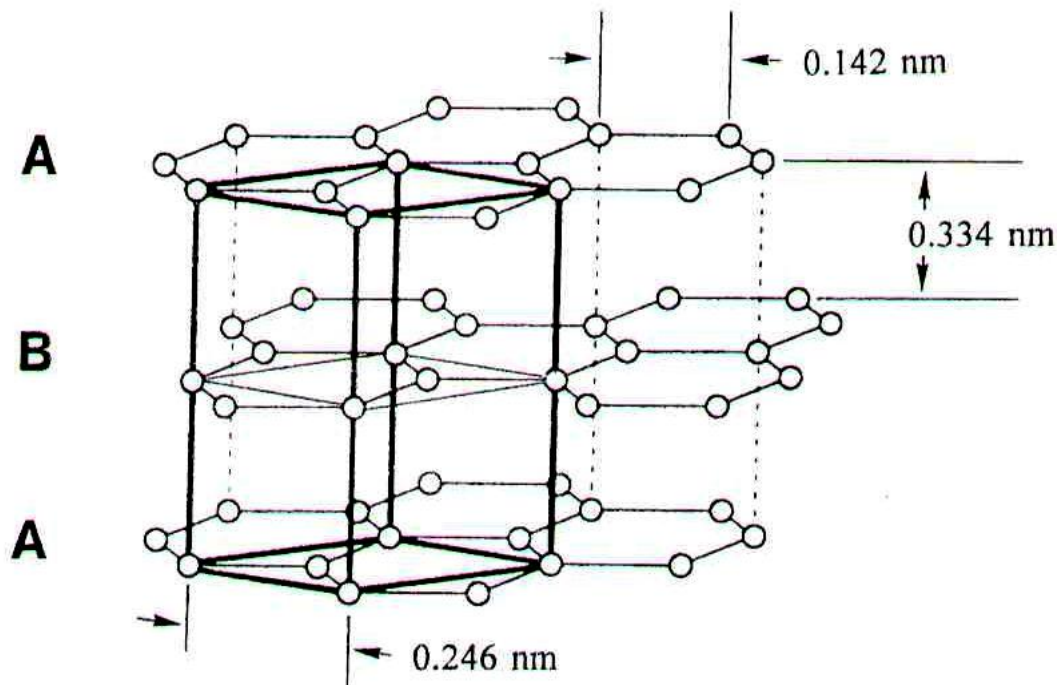
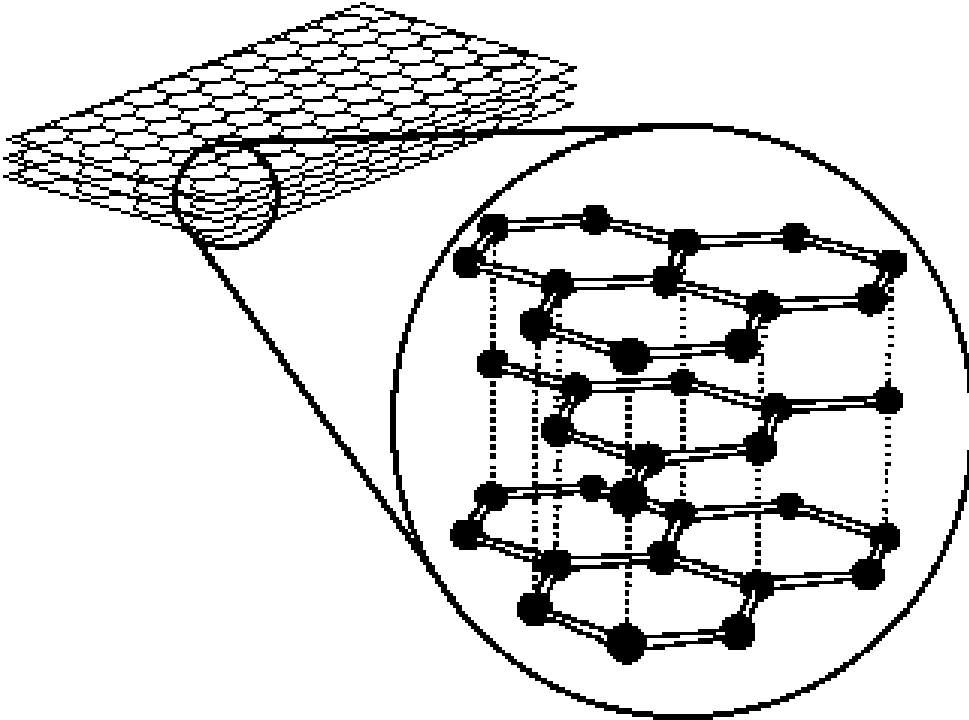
Graphite and diamond



C_{60}



1985



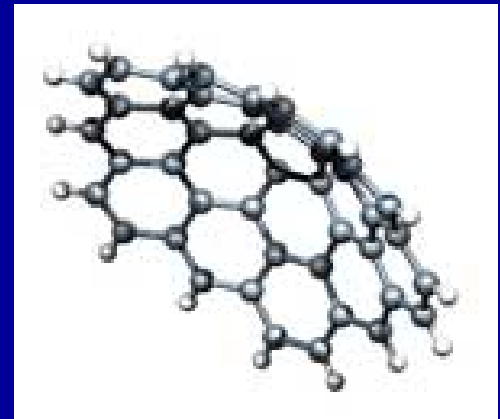
**Crystalline
structure of
graphite**

D.E.H. Jones, *New Scientist* 32, 245 (3 November 1966).

**D. E. H. Jones, *The Inventions of Daedalus* (W. H. Freeman,
Oxford, 1982).**



**Implanting of pentagonal defects into a hexagonal
graphene layer might transform this flat layer
into a giant closed-cage molecule of carbon**





Доклады Академии наук СССР
1973, Том 259, № 3

УДК 541.124

ХИМИЯ

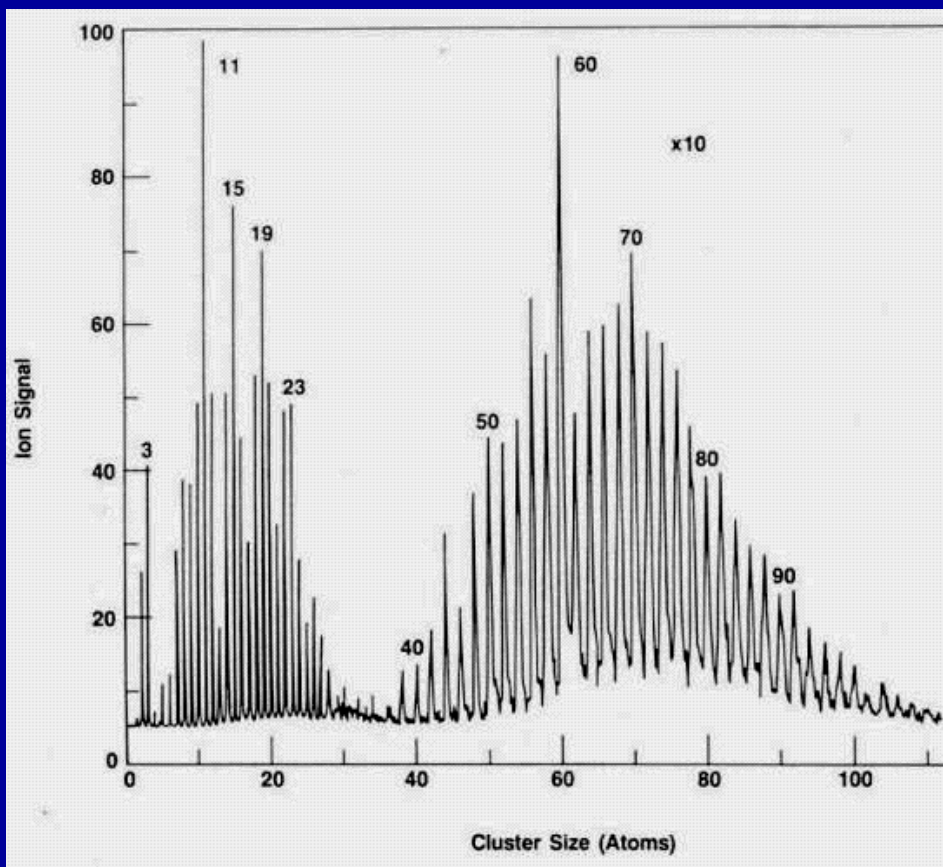
Л. А. БОЧВАР, В. Г. ГАЛ'ПЕРН

О ГЕИОТЕТРАЭДРИЧЕСКИХ СИСТЕМАХ КАРБОДОДЕКАДРЕ,
Э-ПНОСАДРАИТЕ И КАРБО-А-ПНОСАДРЕ

(Представлено академиком А. Н. Пиримановым 26 VI 1972)

Среди полициклических азотсодержащих углеводородов (карбогидрокар-
бидов) особое место занимают карбогидрокарбиды с тетраэдрической
системой связей. Это тетраэдрические углеводороды $C_{12}H_{12}$,
углеводороды с тетраэдрической системой связей, карбогидрокарбиды с тетраэдрической
системой связей (карбогидрокарбиды), имеющие 12, 20, 30, 40, 50, 60, 70, 80, 90, 100, 110, 120, 130, 140, 150, 160, 170, 180, 190, 200, 210, 220, 230, 240, 250, 260, 270, 280, 290, 300, 310, 320, 330, 340, 350, 360, 370, 380, 390, 400, 410, 420, 430, 440, 450, 460, 470, 480, 490, 500, 510, 520, 530, 540, 550, 560, 570, 580, 590, 600, 610, 620, 630, 640, 650, 660, 670, 680, 690, 700, 710, 720, 730, 740, 750, 760, 770, 780, 790, 800, 810, 820, 830, 840, 850, 860, 870, 880, 890, 900, 910, 920, 930, 940, 950, 960, 970, 980, 990, 1000, 1010, 1020, 1030, 1040, 1050, 1060, 1070, 1080, 1090, 1100, 1110, 1120, 1130, 1140, 1150, 1160, 1170, 1180, 1190, 1200, 1210, 1220, 1230, 1240, 1250, 1260, 1270, 1280, 1290, 1300, 1310, 1320, 1330, 1340, 1350, 1360, 1370, 1380, 1390, 1400, 1410, 1420, 1430, 1440, 1450, 1460, 1470, 1480, 1490, 1500, 1510, 1520, 1530, 1540, 1550, 1560, 1570, 1580, 1590, 1600, 1610, 1620, 1630, 1640, 1650, 1660, 1670, 1680, 1690, 1700, 1710, 1720, 1730, 1740, 1750, 1760, 1770, 1780, 1790, 1800, 1810, 1820, 1830, 1840, 1850, 1860, 1870, 1880, 1890, 1900, 1910, 1920, 1930, 1940, 1950, 1960, 1970, 1980, 1990, 2000, 2010, 2020, 2030, 2040, 2050, 2060, 2070, 2080, 2090, 2100, 2110, 2120, 2130, 2140, 2150, 2160, 2170, 2180, 2190, 2200, 2210, 2220, 2230, 2240, 2250, 2260, 2270, 2280, 2290, 2300, 2310, 2320, 2330, 2340, 2350, 2360, 2370, 2380, 2390, 2400, 2410, 2420, 2430, 2440, 2450, 2460, 2470, 2480, 2490, 2500, 2510, 2520, 2530, 2540, 2550, 2560, 2570, 2580, 2590, 2600, 2610, 2620, 2630, 2640, 2650, 2660, 2670, 2680, 2690, 2700, 2710, 2720, 2730, 2740, 2750, 2760, 2770, 2780, 2790, 2800, 2810, 2820, 2830, 2840, 2850, 2860, 2870, 2880, 2890, 2900, 2910, 2920, 2930, 2940, 2950, 2960, 2970, 2980, 2990, 3000, 3010, 3020, 3030, 3040, 3050, 3060, 3070, 3080, 3090, 3100, 3110, 3120, 3130, 3140, 3150, 3160, 3170, 3180, 3190, 3200, 3210, 3220, 3230, 3240, 3250, 3260, 3270, 3280, 3290, 3300, 3310, 3320, 3330, 3340, 3350, 3360, 3370, 3380, 3390, 3400, 3410, 3420, 3430, 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**E.A. Rohlfing, D.M. Cox, A.
Kaldor, J. Chem. Phys. 81,
3322 (1984).**

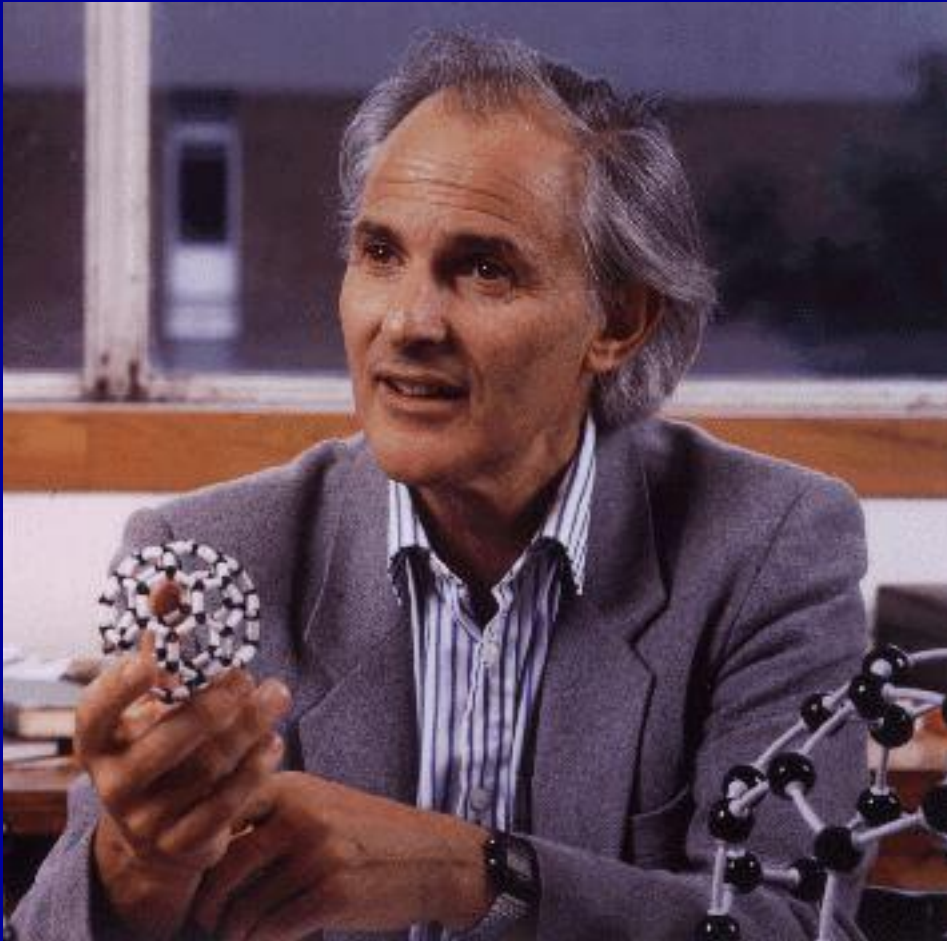


Three distinct regions:

(1) $n < 25$ - small clusters, consisting of the chains and monocyclic rings (well known from the earlier studies);

(2) $25 < n < 35$ -"forbidden zone";

**(3) $36 < n < 150$ –
even-numbered clusters (fullerenes)**

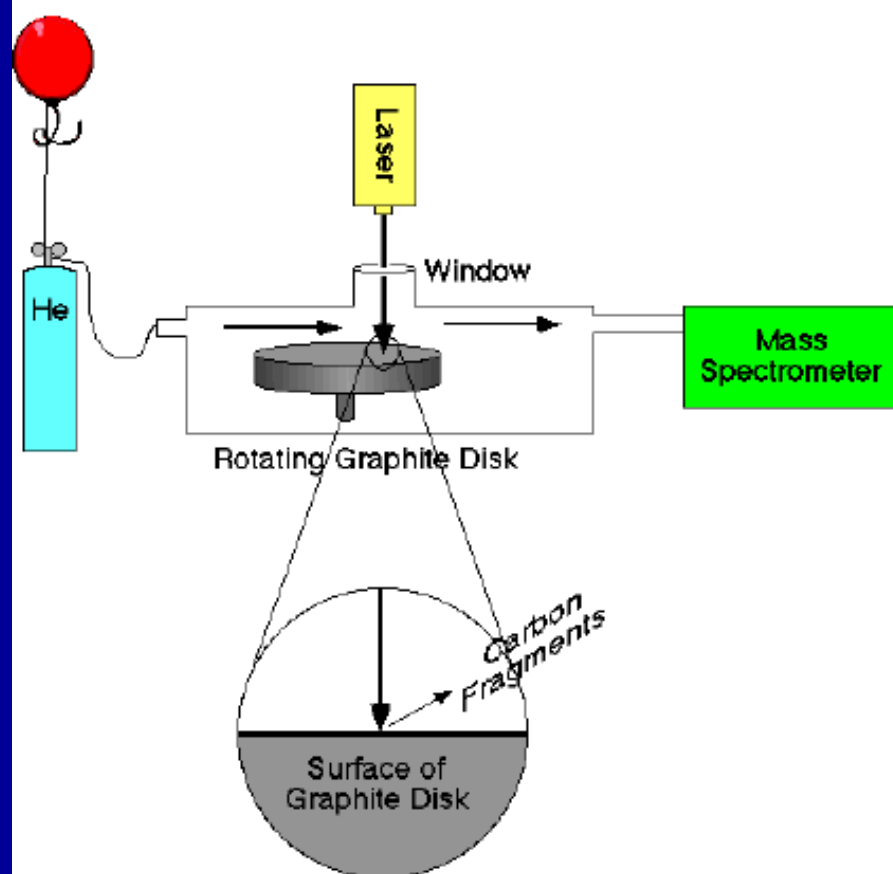


Harold Kroto

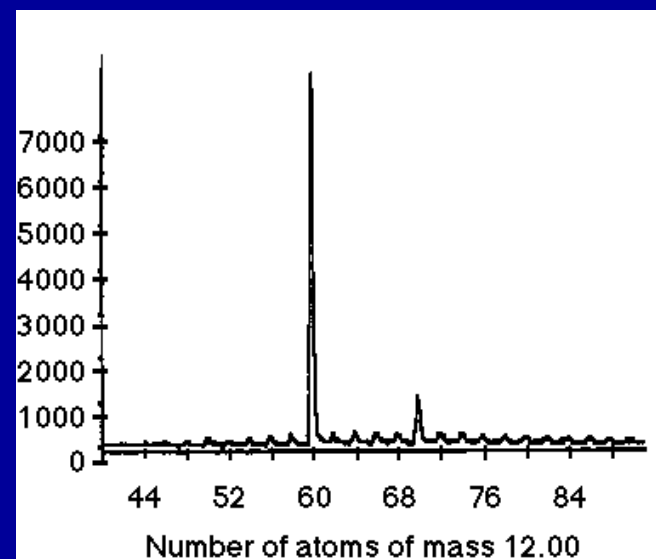


Richard E. Smalley (1943-2005)

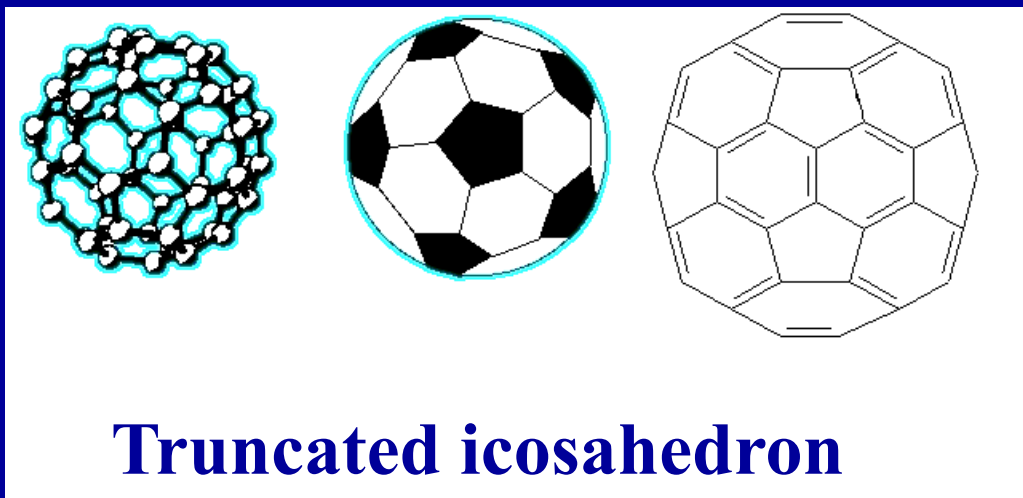
Jerusalem, 1995



September 1985,
Rice University



Structure of C_{60}



12 pentagons;
20 hexagons

$D = 7.1 \text{ \AA}$

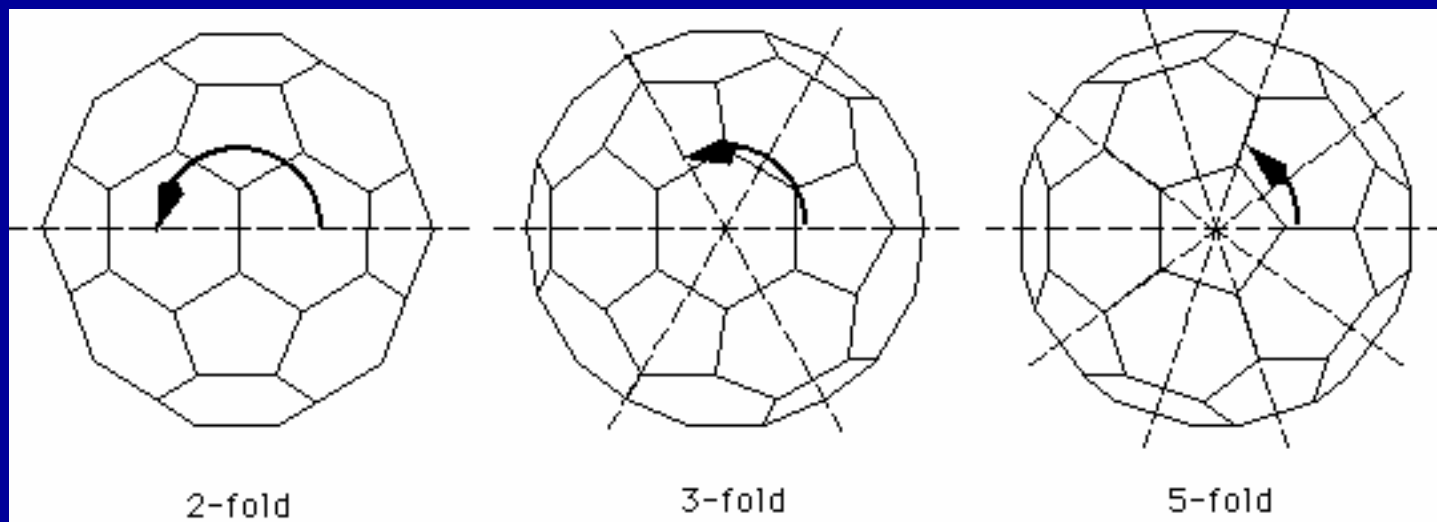
H.W. Kroto, J.R. Heath, S.C.
O'Brien, R.F. Curl and R.E.
Smally, *Nature* 318, 162 (1985).

Single bond – 1.45 \AA ;
Double bond – 1.40 \AA

Kroto, Curl and Smally – Nobel Prize in chemistry, 1996

“The story of C_{60} cannot be recounted without reference to its beauty which results from the incredible symmetry. Another important aspect of the molecule’s aura lies in the name *buckminsterfullerene* and the direct association with the geodesic domes designed by Buckminster Fuller. It invests this elegant molecule with a charisma that has fascinated scientists, delighted lay people and has infected children with a new enthusiasm for science.”

H.W. Kroto, Nobel Lecture, December 7, 1996



15 rotation axes (180°) 10 rotation axes (120°) 6 rotation axes (72°)

“These groups (I , I_h) are of no physical interest since they have not realized themselves as symmetry groups of molecules”

L. Landau and E. Lifshits, Quantum Mechanics, 1948.

2. Buckminster Fuller's energetic geometry and geodesic domes

Richard Buckminster Fuller (1895-1983) —

architect, philosopher, engineer, inventor, author, cartographer, geometrician, futurist, teacher and poet... one of the most original thinkers of the second half of XX century (*Encyclopedia Britanica*)



1927, the aim: a non-profit search for design patterns that could maximize the social uses of the world energy and industrial resources

Richard Buckminster Fuller (1895-1983) —

architect, philosopher, engineer, inventor, author, cartographer, geometrician, futurist, teacher and poet... one of the most original thinkers of the second half of XX century (*Encyclopedia Britanica*)

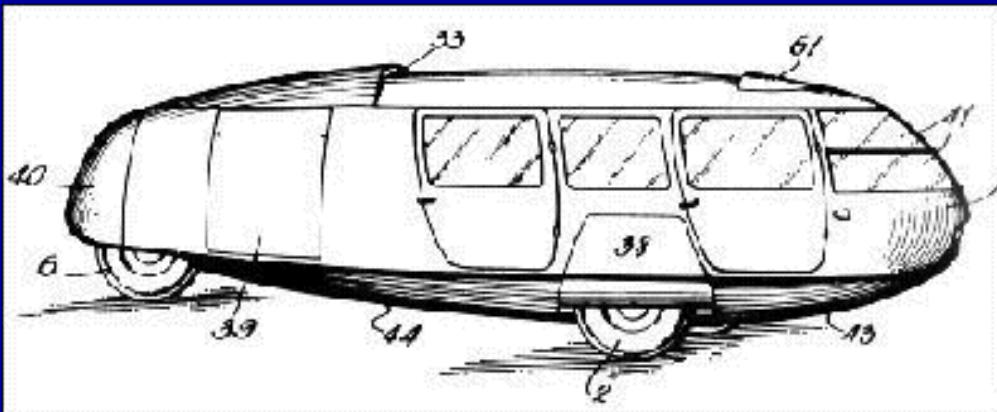


*Dymaxion House,
1928. Wichita, Texas*



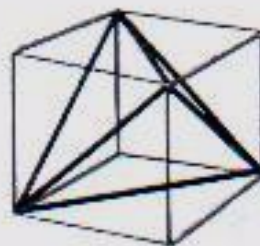
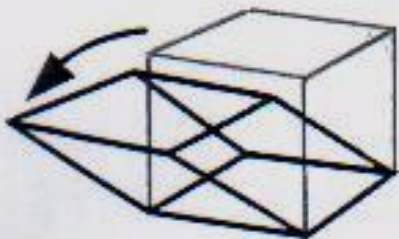
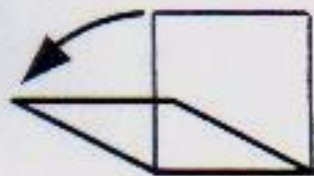
1927, the aim: a non-profit search for design patterns that could maximize the social uses of the world energy and industrial resources

Dymaxion Car, 1928-1933



190 km/hour, 12 passengers

Energetic-Synergetic Geometry



A

B

C

Unstable shapes
Square and
Cube

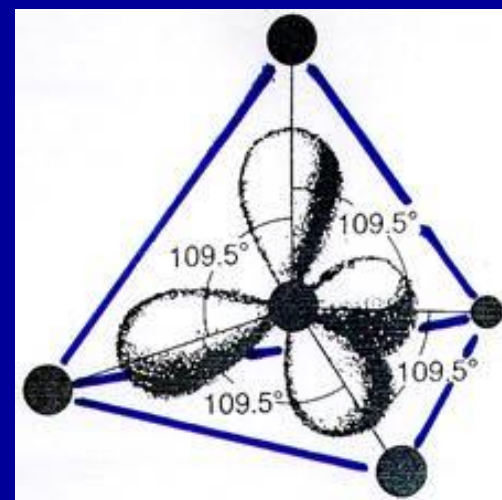
Stable shapes
Triangle and
Tetrahedron

Stable shapes
support
the unstable ones

**Tetrahedral configuration of
carbon bonds**

Jacobus H. van't Hoff, 1874

First Nobel Price in Chemistry, 1901



Square and cube (A) have no inherent structural strength – they can collapse in any direction. The triangle and tetrahedron (B) have built-in strength and stability. Column (C) shows how square and cubes become stable *only* when turned into triangles and tetrahedra.

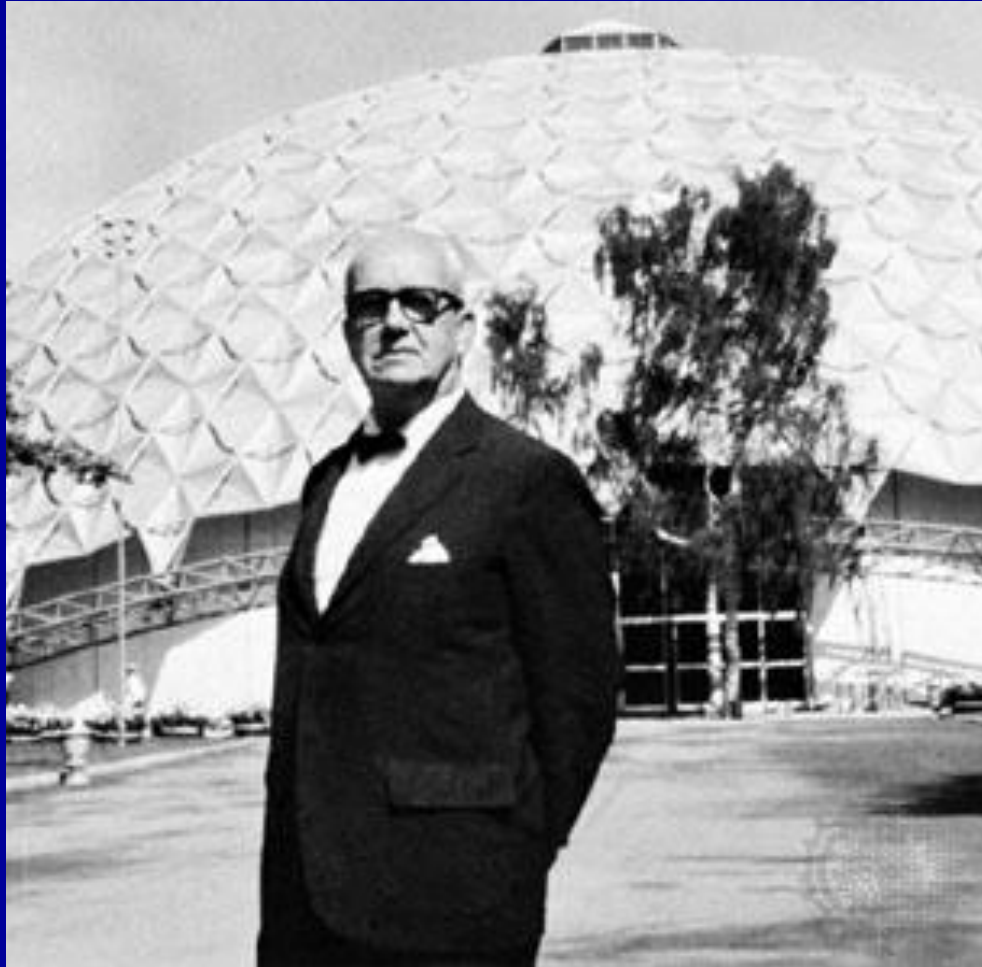
1954 - the first Bucky's patent on geodesic domes;

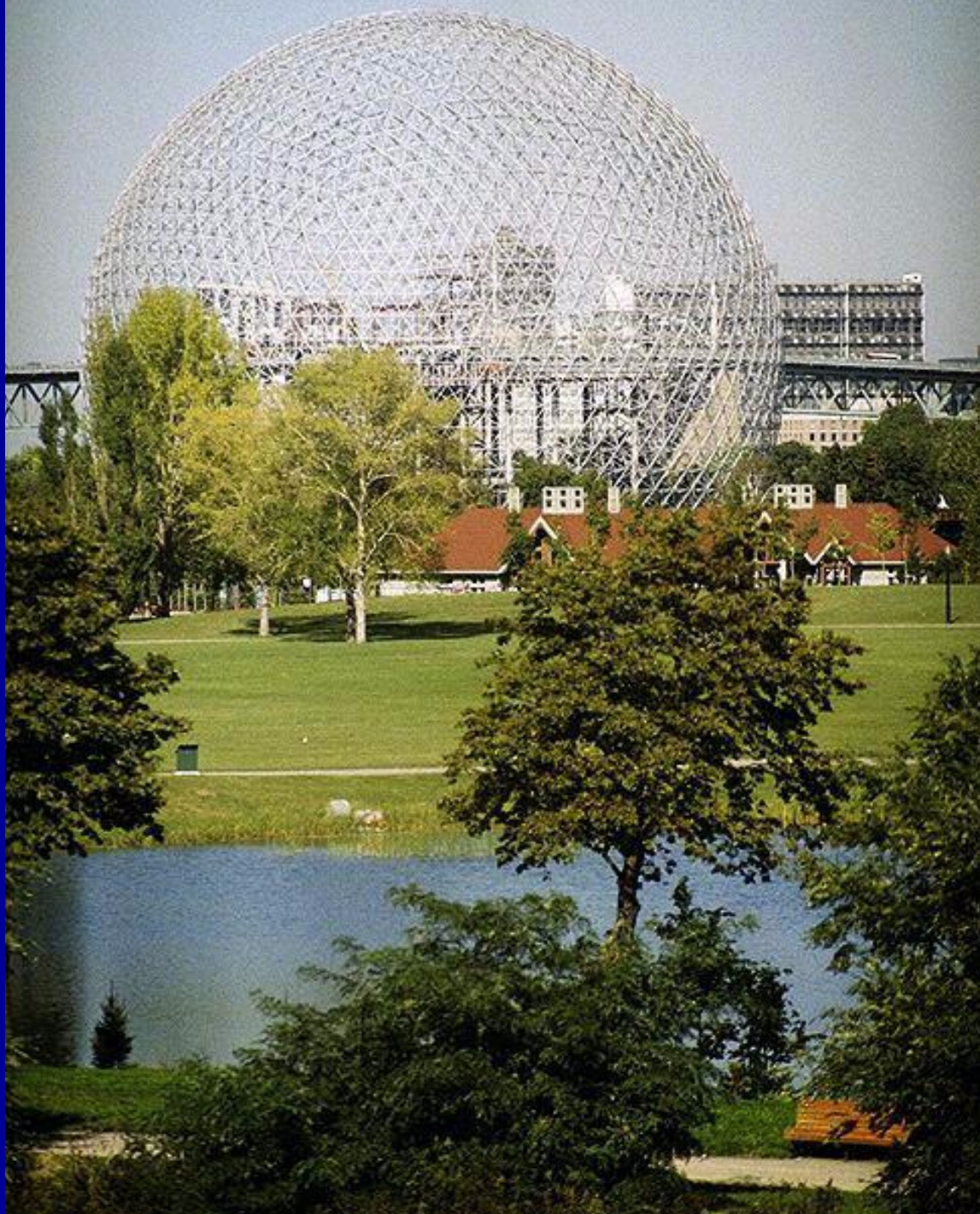
**1958 - geodesic dome for the Union Tank Car Company
(Los Angeles), 117 m diameter and 35 m height;**

**1958-1959 - Headquarter of the “ASM International”, the
Materials Information Society (Cleveland, Ohio)**



1959 – Pavilion of the first US exhibition in Moscow





**1967 –
the US Pavilion at
Expo'67,
Montreal, Canada**

Geodesic domes (definitions & statements)

Geodesic sphere is a polyhedral structure could be effectively inscribed in a sphere.

Geodesic domes are fractional parts of complete *geodesic spheres*. Actual structures range from less than 5% to 100% (a full sphere).

A sphere is already efficient: it encloses *the most volume with the least surface*. Thus, any dome that is a portion of a sphere has the least surface through which to lose heat or intercept potentially damaging winds.



GEOD, Paris. Photo by E. Katz

A geodesic dome uses a pattern of self-bracing triangles that gives maximum strength and stability using the least material possible.

Frequency of a dome relates to the number of smaller triangles into which it is subdivided. A high frequency dome has more triangular components and is more smoothly curved and sphere-like

Geodesic domes get stronger, lighter and cheaper per unit of volume as their frequency and size increase - just the opposite of conventional building



'Garden of Eden', Cornwall, UK, 2001

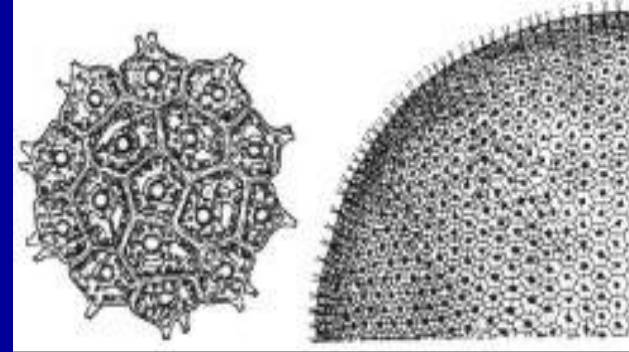
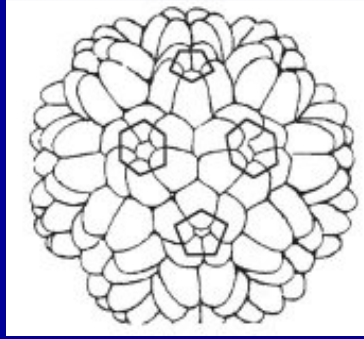
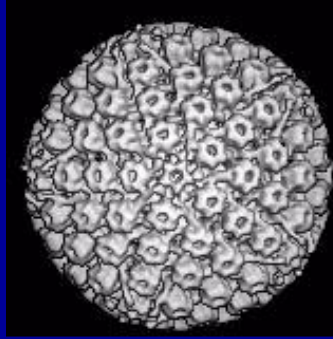




Photo by E. Katz

Fullerene-like viruses

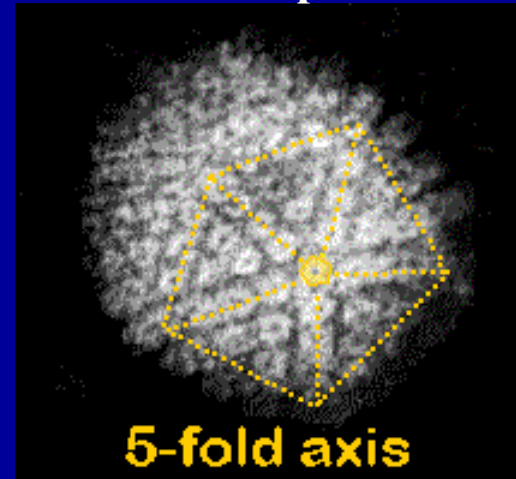
(small geodesic domes with icosahedral symmetry)



20-300 nm

Herpes

D.L.D. Caspar and A. Klug. Physical Principles in the Construction of Regular Viruses, in Cold Spring Harbor Symp. Quant. Biol. V.27, 1 (1962):

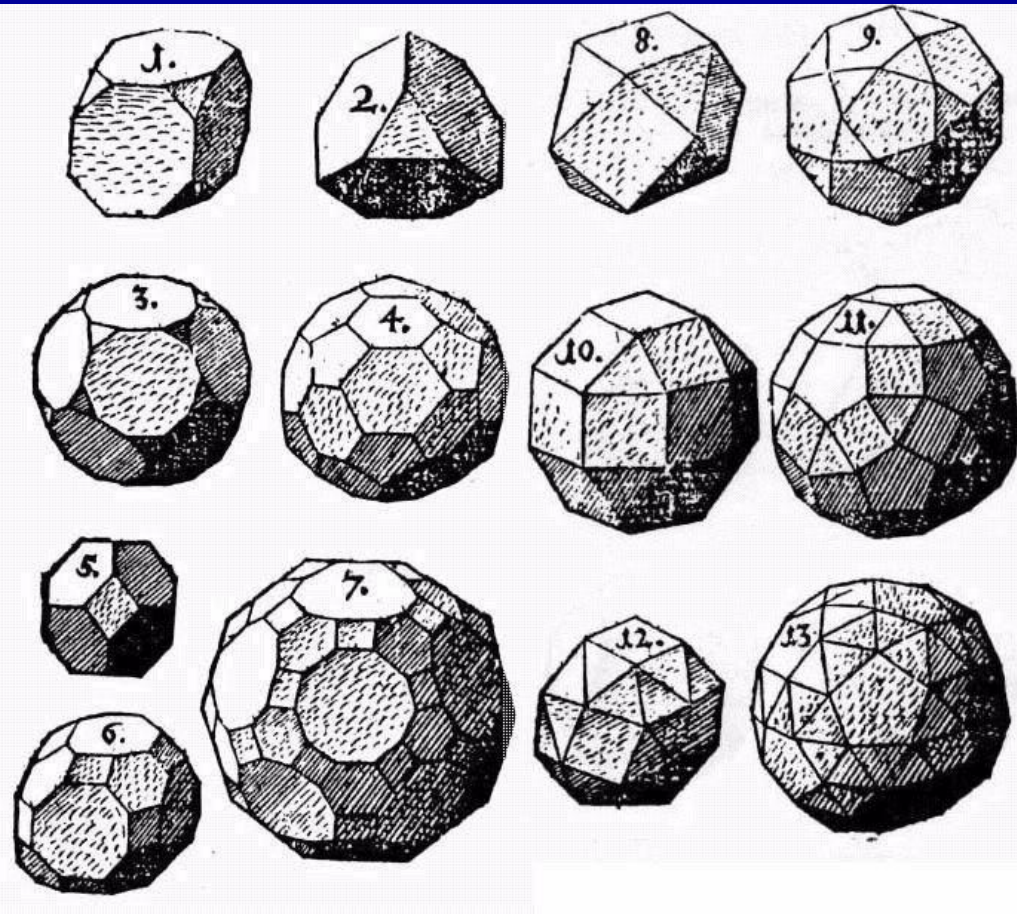


5-fold axis

“The solution we have found was, in fact, inspired by the geometrical principles applied by Buckminster Fuller in the construction of geodesic domes and his physically oriented geometry of efficient design.”

3. Long history of exploration of polyhedra: science and fine art

Archimedes (287-212 b.c.): first researcher of truncated icosahedron



Archimedian polyhedra or archimedean solids:
polyhedra with faces as regular polygons and vertices located in equivalent positions (as carbon atoms in the C_{60} molecule).

Archimedean solids consist of at least 2 different types of polygons (that makes them different from regular polyhedra or *platonic solids*)

From Johannes Kepler's "Harmonice Mundi" ("The Harmony of the World"),
1619

Platonic solids (regular polyhedra)

Plato (~420 – 347 b.c.),
probably,
Pythagoreans

Hipparchus (? – 127 b.c.)

Tetrahedron -

Cube -

Octahedron -

Icosahedron -

Dodecahedron -

Classical elements:

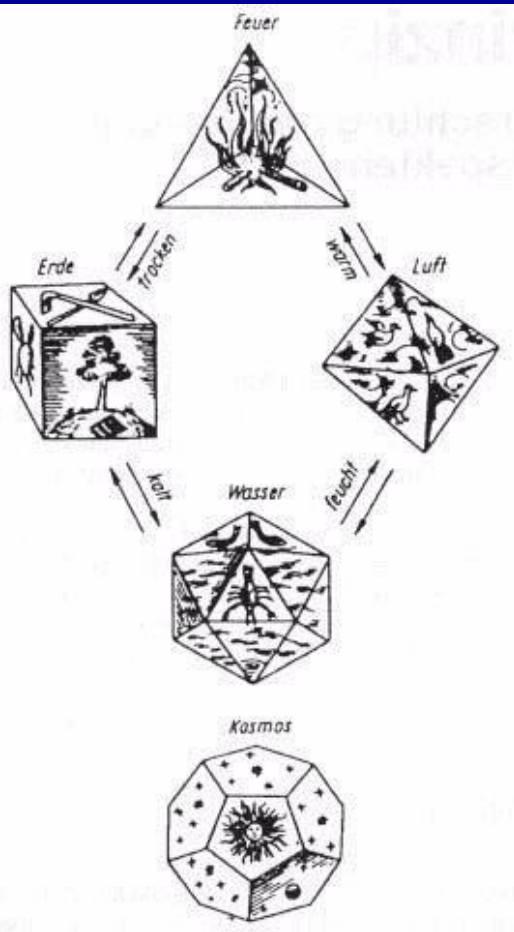
fire,

earth,

air,

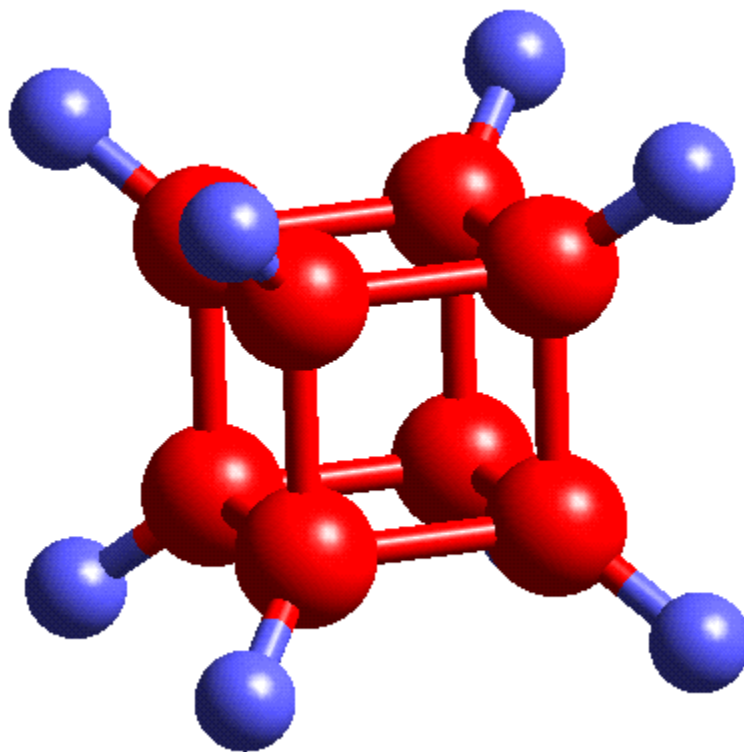
water,

the universe



From Johannes Kepler's
“Harmonice Mundi”
(“The Harmony of the
World”), 1619

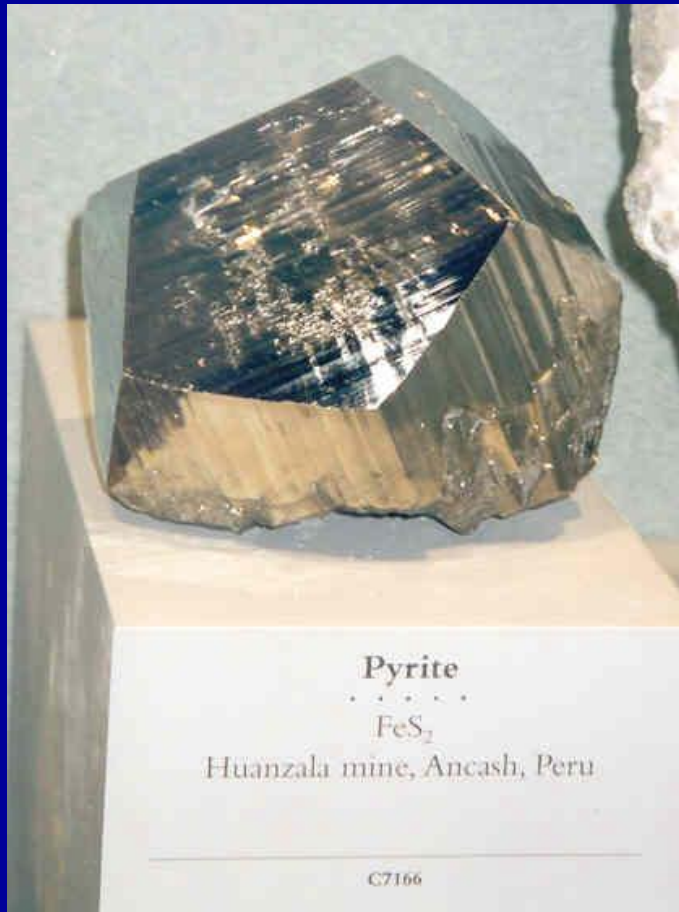
“Forbidden” chemical compound –
Cubane – C_8H_8 (1964)



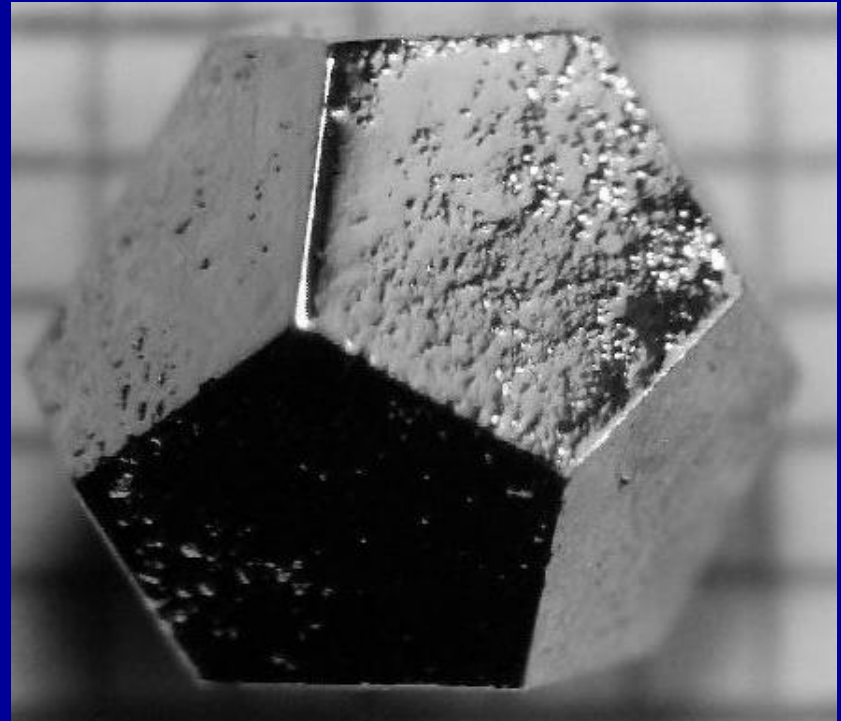


**Quasi-Octahedron. Louvre, Paris.
Photo by E. Katz**

Dodecahedron



**Natural History Museum,
Washington, D. C.
Photo by E. Katz**

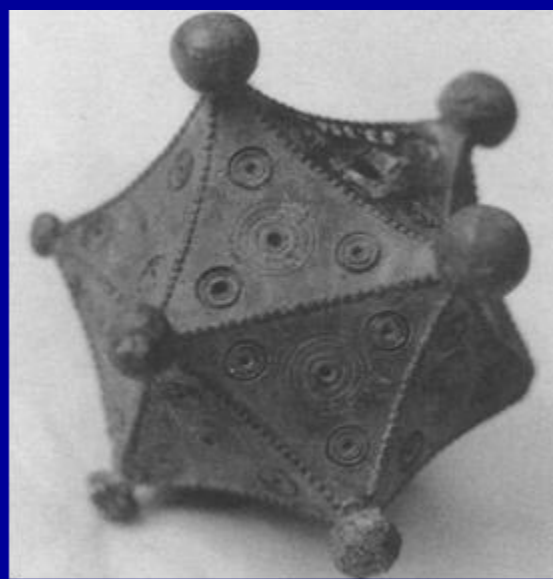


Quasicrystal of a Ho-Mg-Zn alloy.

Photo from Wikipedia.



**Sundial by John Mylne (1633).
Courtyard of the Palace of
Holyroodhouse, Edinburgh.
Photo by E. Katz**



**Bronze icosahedron
(about 100-300 A. D.)
excavated in the Roman
provinces along the
Rhine river. Photo from
[B. Artmann, The
American Mathematical
Monthly, 18, 132 (1996)].**

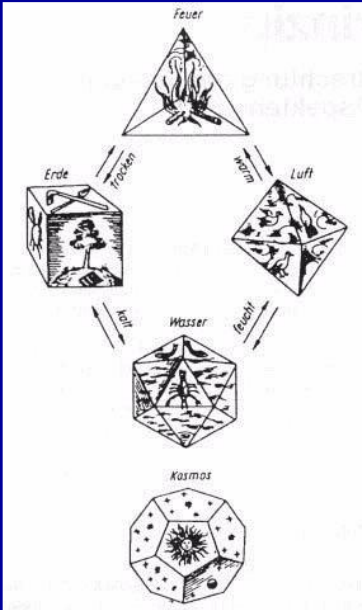


**Nicolas Dings. Monument to
Baruch de Spinoza (2008).
Amsterdam. Photo by E. Katz**

Characteristics of platonic solids

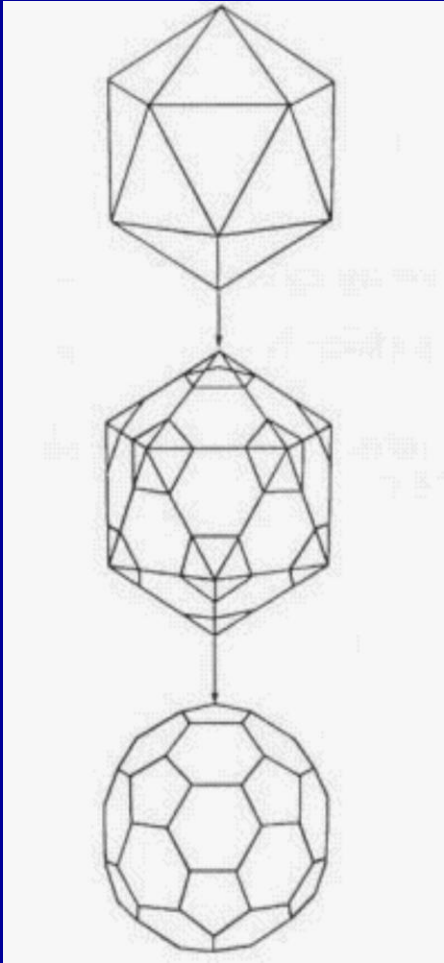
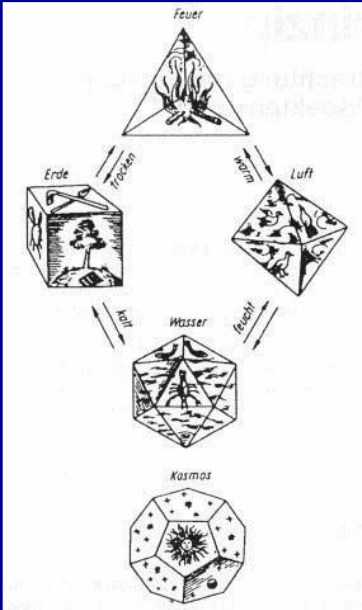
Polyhedron	Edges in each face, m	Edges that connect in each vertice, n	Faces, F	Edges, E	Vertices, V	$F-E+V$
Tetrahedron	3	3	4	6	4	2
Cube	4	3	6	12	8	2
Octahedron	3	4	8	12	6	2
Icosahedron	3	5	20	30	12	2
Dodecahedron	5	3	12	30	20	2

Duality:
Cube and octahedron are dual to each other; icosahedron is dual to dodecahedron (and vice versa) and tetrahedron is dual to itself.



Characteristics of platonic solids

Polyhedron	Edges in each face, m	Edges that connect in each vertice, n	Faces, F	Edges, E	Vertices, V	$F-E+V$
Tetrahedron	3	3	4	6	4	2
Cube	4	3	6	12	8	2
Octahedron	3	4	8	12	6	2
Icosahedron	3	5	20	30	12	2
Dodecahedron	5	3	12	30	20	2

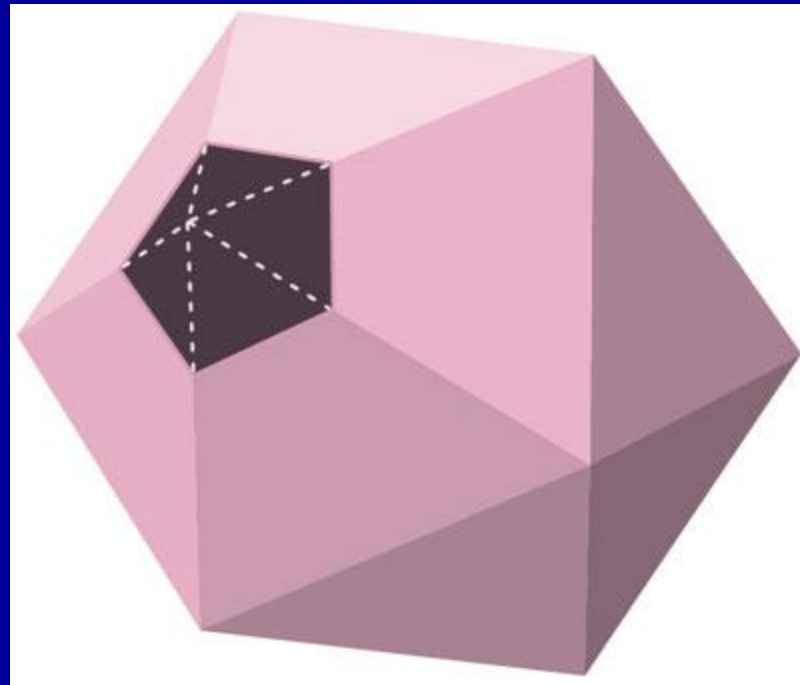


https://www.americanscientist.org/article/the-topology-and-combinatorics-of-soccer-balls?fbclid=IwY2xjawFp-DNleHRuA2FlbQIxMQABHa0zUKxl-HwHlhFeivM-gl8IHrEcKv5Mz6bUDGdIreciug9PaqcRsMhp4Q_aem_FQWyq48fQpw2zn3qdWgDqw

American Scientist

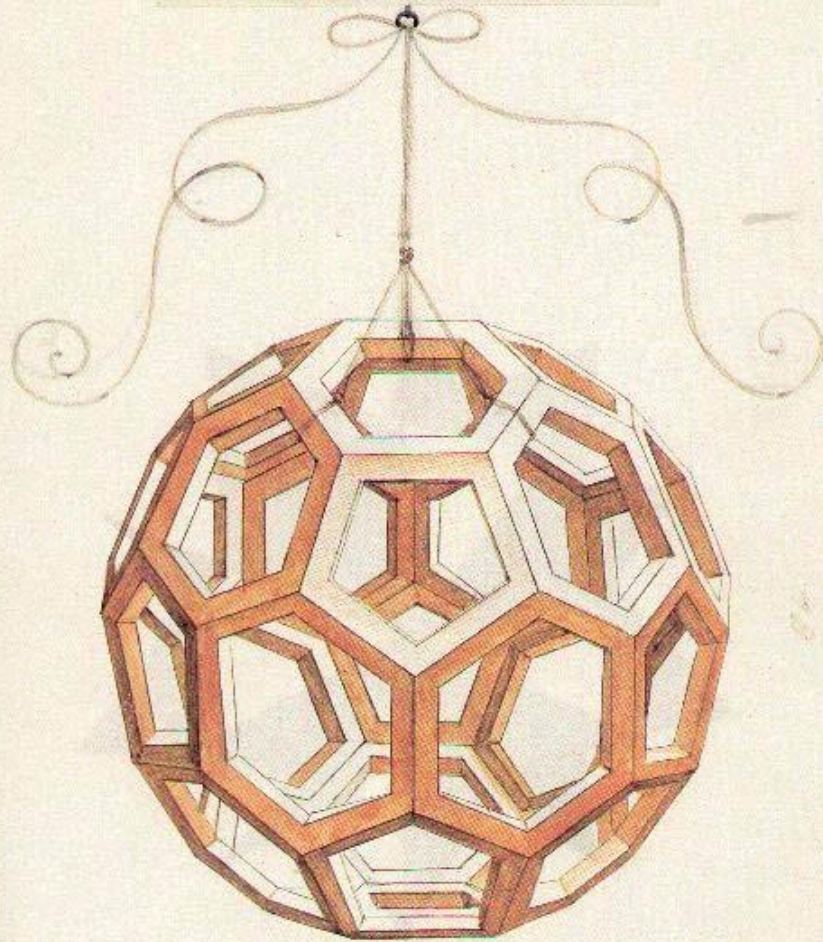
The Topology and Combinatorics of Soccer Balls

By Dieter Kotschick



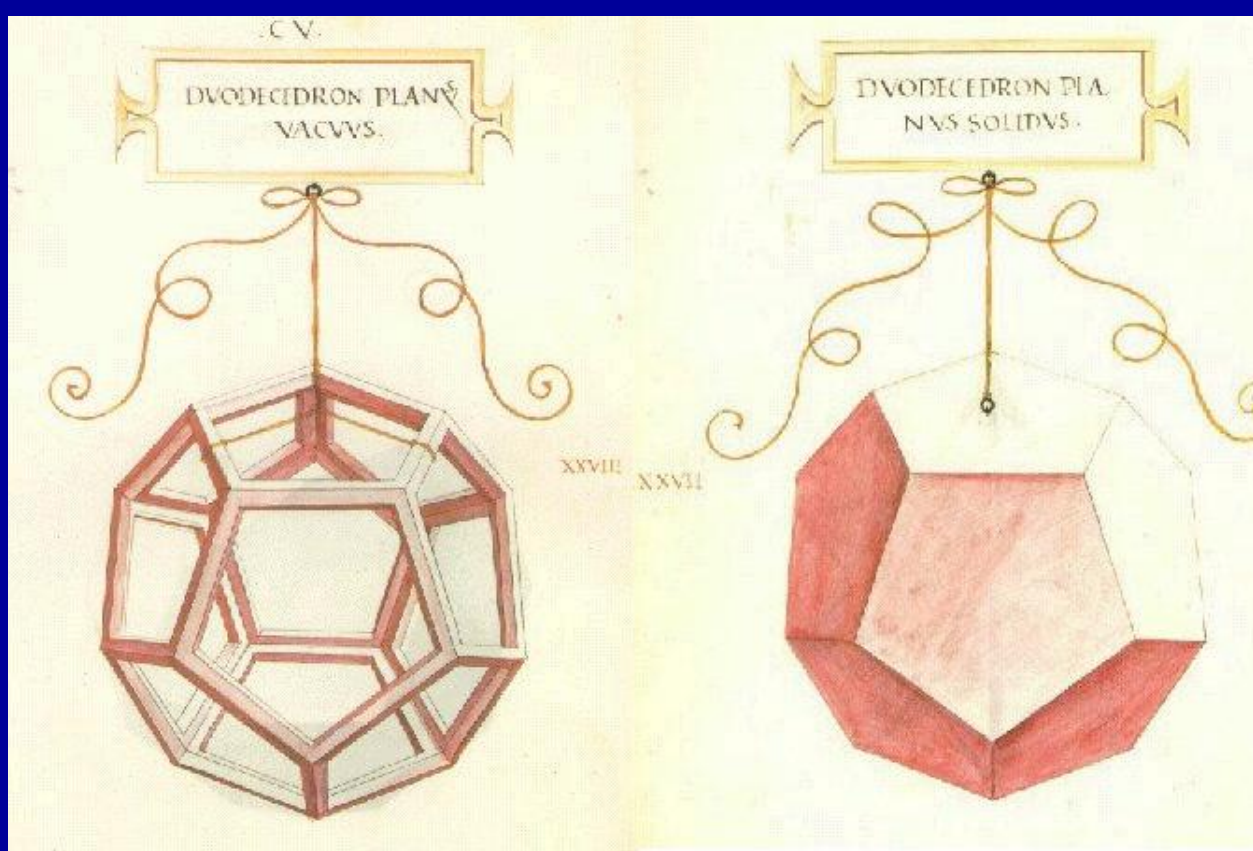
CIII.

YCOCEDRON · ABSCISVS
VACVVS.

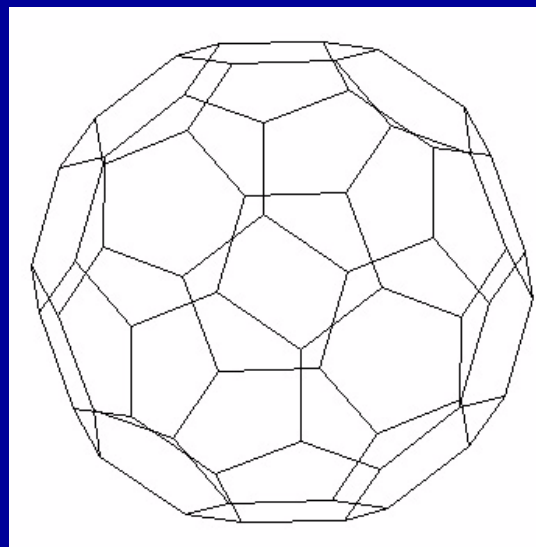


XXIII

Leonardo's drawing of truncated icosahedron from Luca Pacioli's book "The Devine Proportion" ("De Devina Proportione"), 1509



**Leonardo's drawing
of dodecahedron by
a method of
solid segments
and *solid faces*
from Luka Pacioli's
book "The Devine
Proportion" ("De
Devina Proportione"),
1509**



Luca Pacioli, 1445-1517

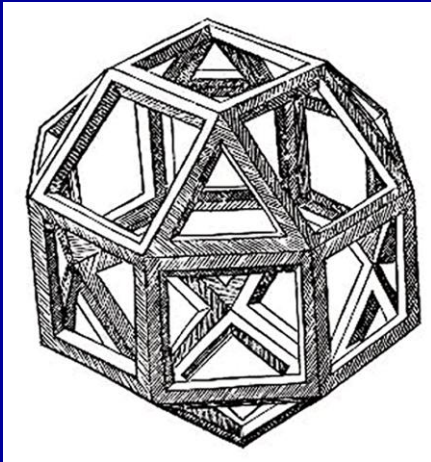
Rombicuboctahedron



Jacopo de Barbari. Luca Pacioli's portrait, 1495

Rhombicuboctahedron.

(a) Leonardo's printing in "The Devine Proportion" (1509). (b) National Library of Republic of Belarus in Minsk (2006, architects Mihail Vinogradov and Viktor Kramarenko). Photo by Galya Katz.



Luca Pacioli, 1445-1517

Rombicuboctahedron

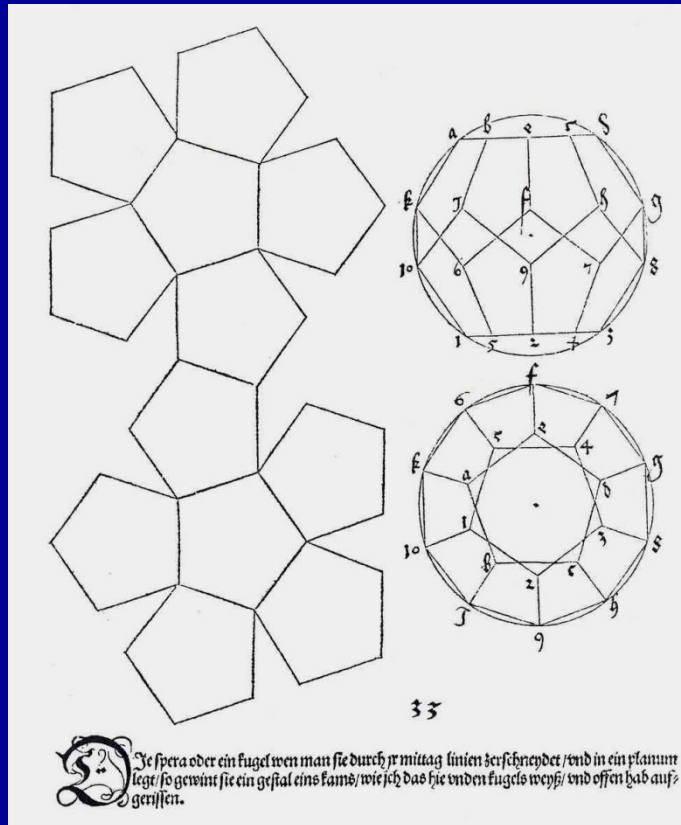


Jacopo de Barbari. Luca Pacioli's portrait, 1495

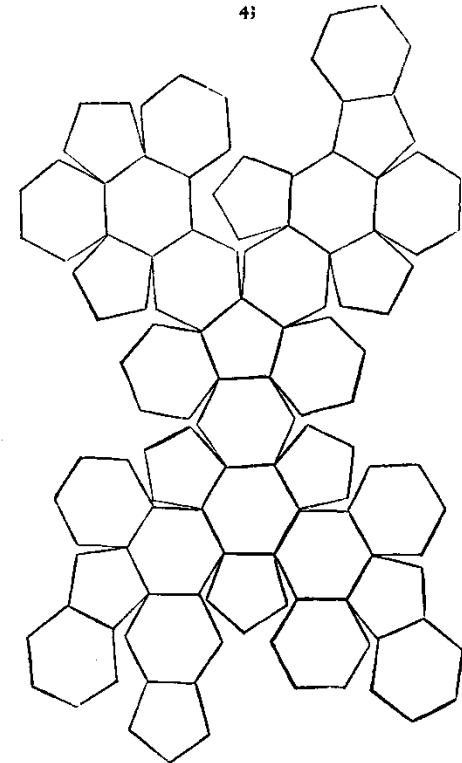


Self-portrait, 1498

Albrecht Durer, 1471-1528



In anders das mach auß zweinsig sechsecker flachen seiden / gleichseitig vnd windlich /
so man darzu thut zwölff fünfecker flacher seiden / so die gleichseitig gegen den sechseckem
seiden sind / vnd in jnen selbe auch gleich windlich vnd ebenlich an einander gesetzt wer
den / wie ich das offen im plano hernach hab außgerissen / So man dann das alles zusam
menleget / so wirt ein corpus daraus / das gewintet zwep vnd sechsig eck / vnd neungsig scharffe
seiten / die Corpus rüret in einer hohlen kugel mit allen seiten eben an.

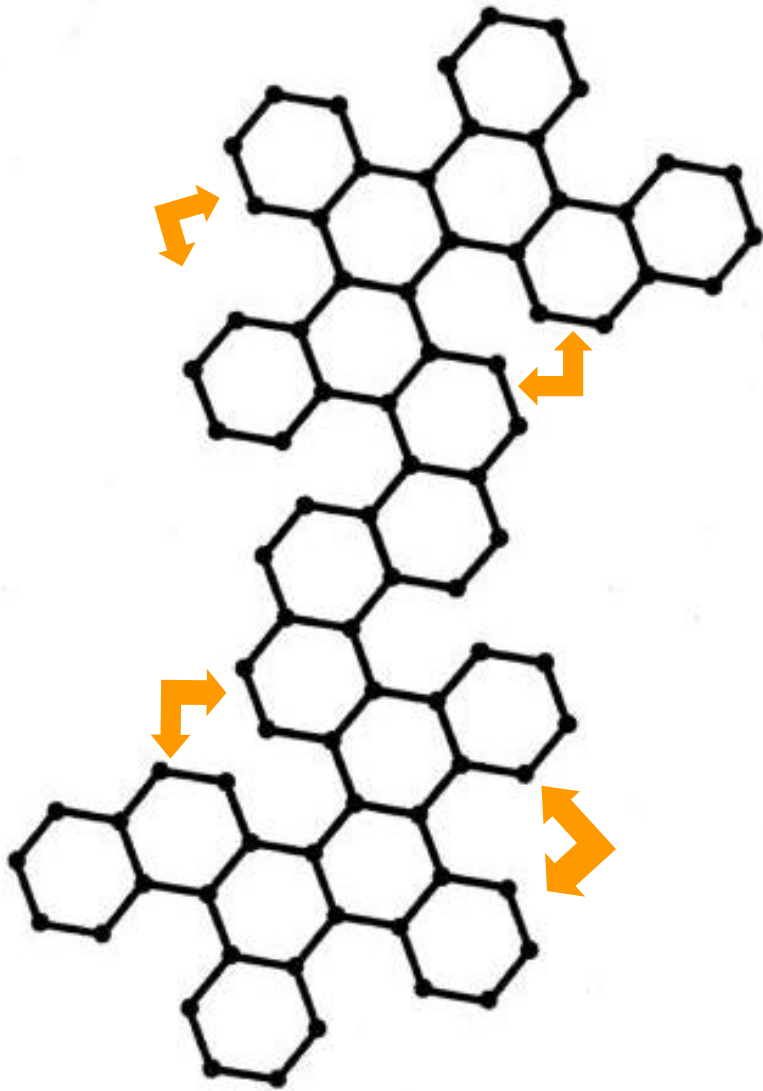


Flat nets of dodecahedron and truncated icosahedron from Albrecht Durer's treatise "Painter's Manual" ("Underweysung der Messung"), 1525.

How to make a Buckyball

Please...

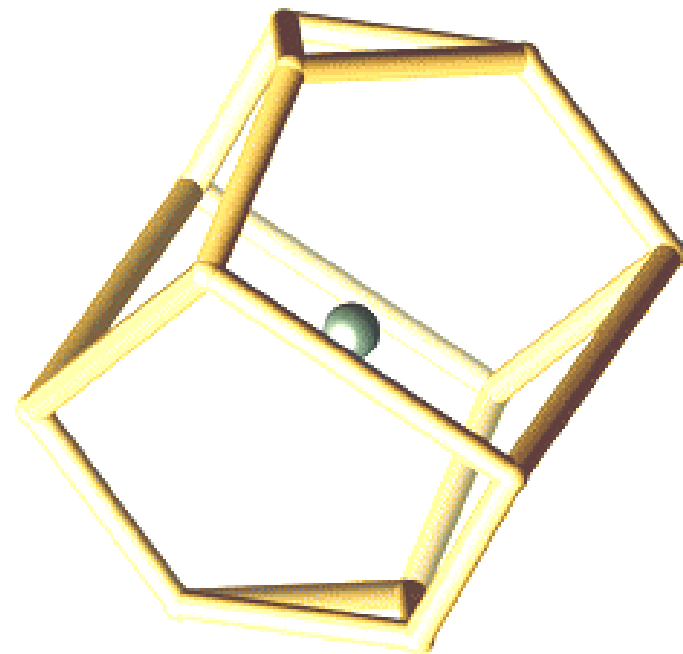
- Copy this net;
- Carefully cut it; -
- Start to curve it along the edges between the adjacent hexagons;
- Bring together each of the four pairs - of edges indicated by arrows and fasten them with small pieces of a transparent tape.



F. Chung and S. Sternberg,
Mathematics and the Buckyball,

American Scientist 81, 56 (1993).

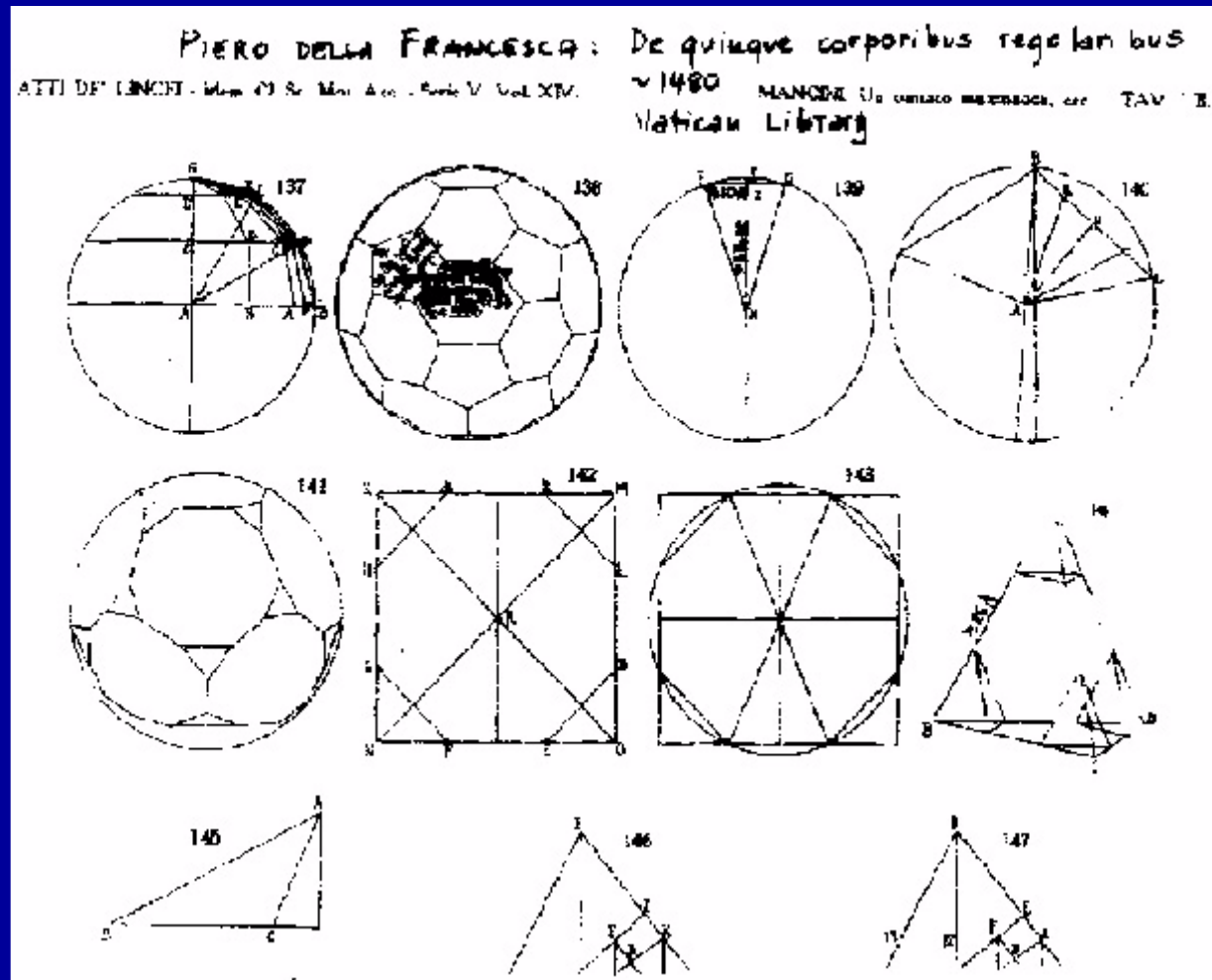
Truncated rhombohedron (Dürer's solid)



NiAs₁₂ cluster

Albrecht Dürer. "Melancholia I" (1514)

Piero della Francesca (1420 - 1492)



The oldest known picture of truncated icosahedron from Piero della Francesca's manuscript "Short book on the five regular solids" ("Libellus de quinque corporibus regularibus"), dated 1480, Vatican Library.



Piero della Francesca. *Montefeltro Altarpiece*, 1465, Pinacoteca di Brera, Milan

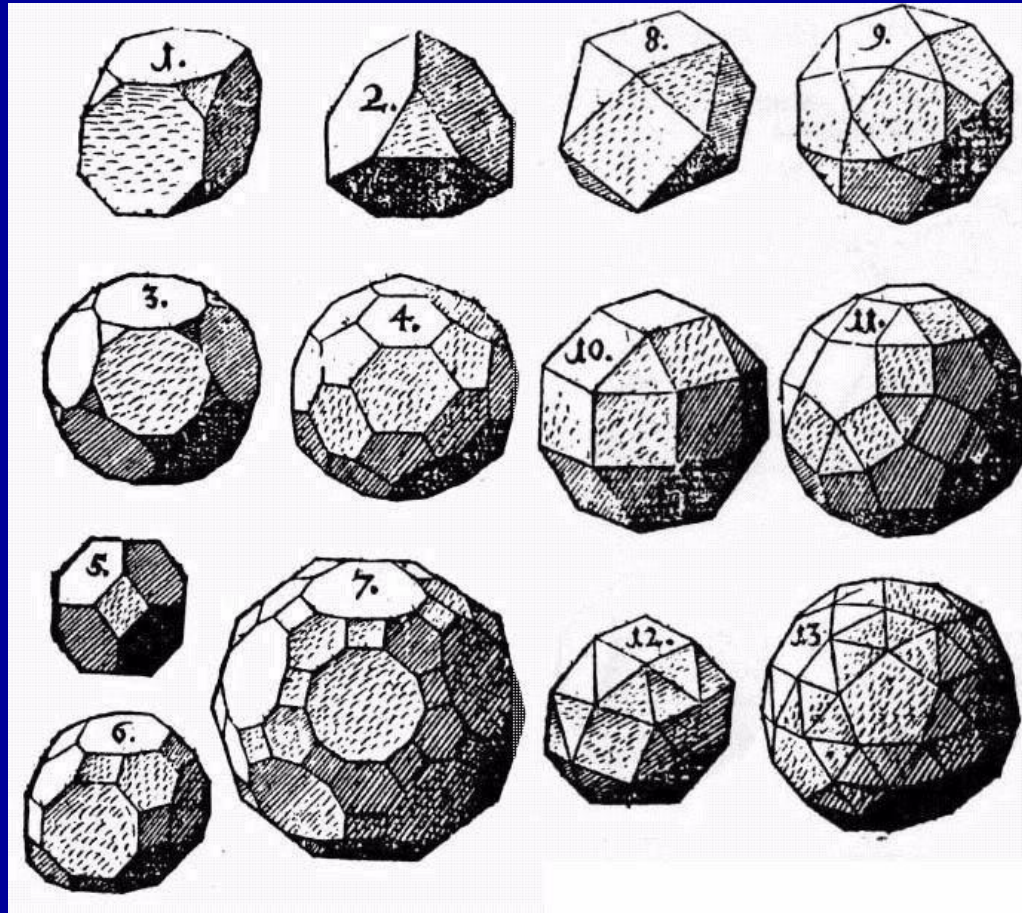


Memorial desks of Piero della Francesca and Luca Pacioli in San Sepolcro. Photos by E. Katz

In honor of Piero della Francesca
in the fifteenth century
the sovereign painter
from whose mastery Perugino
learned the marvels of art
and Italy the geometric principles of
perspective
here where her great son lived
and where at 82 years he ascended
into the heavens
this memorial is placed in the year 1876
by a grateful and reverential country

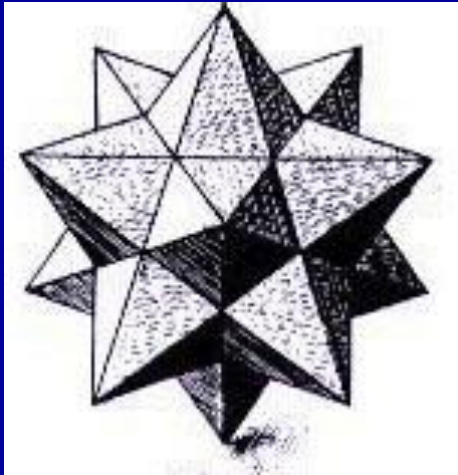
To Luca Pacioli who was friend and consultant
to Leonardo da Vinci
and Leon Battista Alberti
who first gave to algebra the language
and the structure of science
and whose was the great discovery
of applying it to geometry...
The people of San Sepolcro upon
the initiative of the society of workers
ashamed of 320 years of oblivion
to the great fellow citizen
placed [this memorial] in 1878

Archimedian polyhedra or archimedean solids



From Johannes Kepler's "Harmonice Mundi" ("The Harmony of the World"), 1619

Star-like regular polyhedra



Johannes Kepler.
“**Harmonice Mundi**”
 (“**The Harmony of
the World**”),

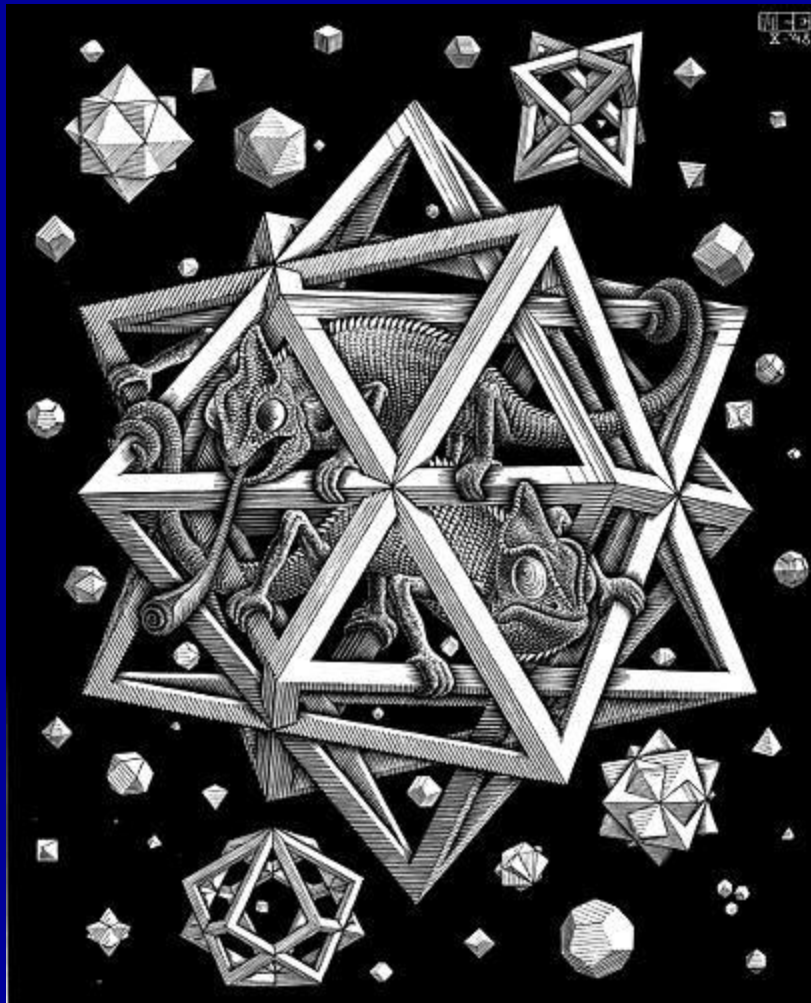
1619



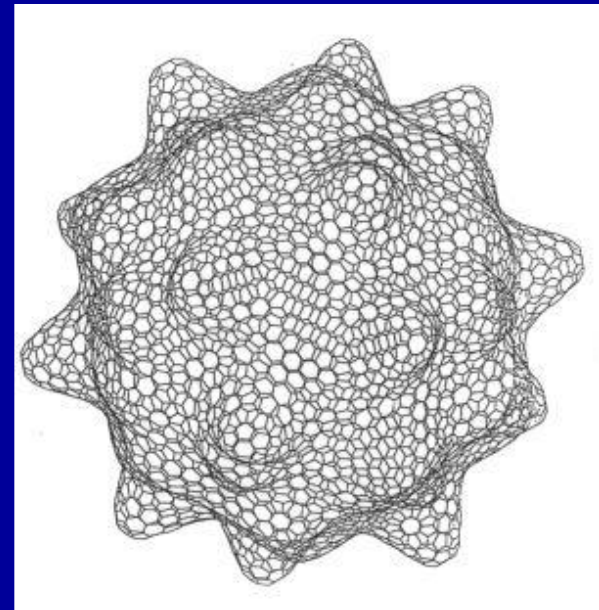
**Paolo Ucello (~
1420's ?). St Marco
Cathedral, Detail
of mosaics**

**Monument by Mimmo
Paladino near to the
entrance to Leonardo
da Vinci Museum in
the village of Vinci
(Leonardo's birthplace,
Toscany). Photo by
E.A. Katz.**





**M.C. Escher. Stars
(1948)**



Star-like carbon cluster

**Intarsia panels by Fra Giovanni da Verona (1457 - 1525)
in Territorial Abbey of Monte Oliveto Maggiore
(near Siena)**



Photo by E. Katz



Salvador Dali. “The Sacrament of the Last Supper (1955). The National Gallery of Art in Washington, D.C.



**Salvador Dalí. “In search of the fourth dimension” (1979).
Fundació Gala-Salvador Dalí, Figueres.**



**Photo by
E. Katz**

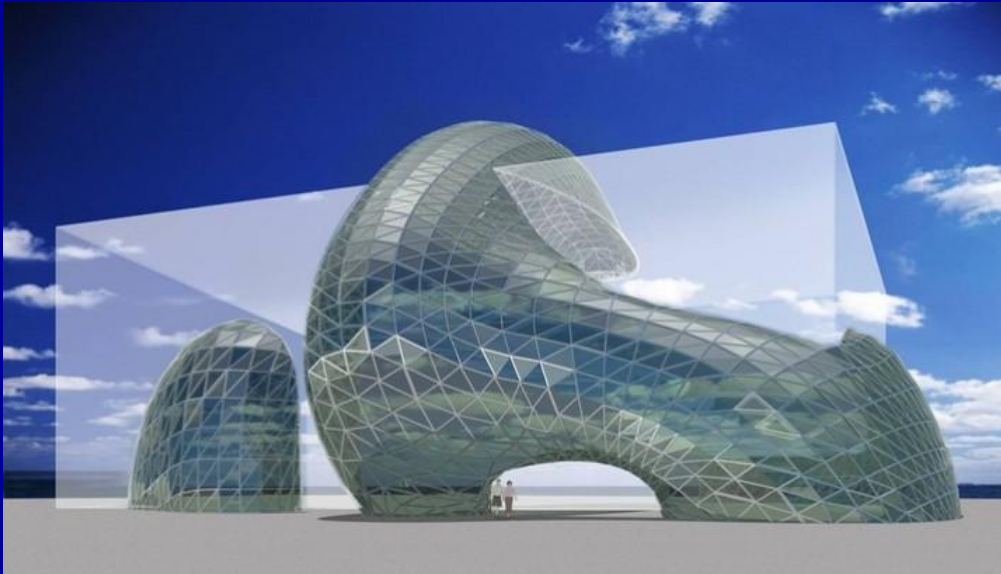
**Dalí House-Museum ,Port Lligat,
Spain**



**Dalí Theatre –Museum, Figueres, Spain
[arch. Emilio Pérez Piñero (1935-1972)]**



Dali Museum, St. Petersburg, FL (architect Yann Weymouth, 2011)



4. Leonhard Euler's theorem

Leonhard Euler (1707 - 1783)

Euler's theorem for convex polyhedra

$$F - E + V = 2$$

where F - faces, E - edges, V -vertices

Descartes (1596-1650)

L. Euler, *Novi corumentarii*
academie Petropolitanae 4, 109 (1752/3).



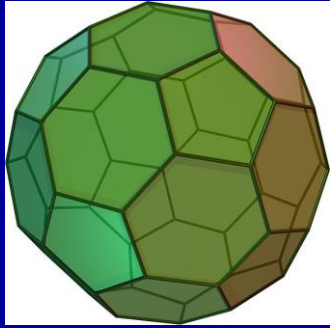
General Euler's theorem

$$F - E + V = n$$

n – Euler's parameter

(n = 0 for torus, n=2 - 2g for polyhedra with g 'through holes')

This is the first theorem of topology, part of mathematics that is concerned with the properties of a geometric object that are preserved under continuous deformations, such as stretching, twisting, crumpling and bending, but not tearing or gluing.



A convex polyhedron (truncated icosahedron) is homeomorphic to a sphere (soccer ball)



A continuous deformation (a type of *homeomorphism*) of a cow into a sphere and a cup into a torus



Henry Segerman. Topological joke. Topological transformations of surfaces with *genus* 1. The dumpling (torus) is constantly transformed into a cup (bullet with 1 handle).

Euler's theorem for convex polyhedra

$$F - E + V = 2$$

where F - faces, E - edges, V -vertices

Fullerenes are closed-cage molecules of pure carbon in the shape of polyhedra with only pentagonal and hexagonal faces

$F = p + h$ where p and h are number of pentagonal and hexagonal faces, respectively

$$2E = 5p + 6h$$

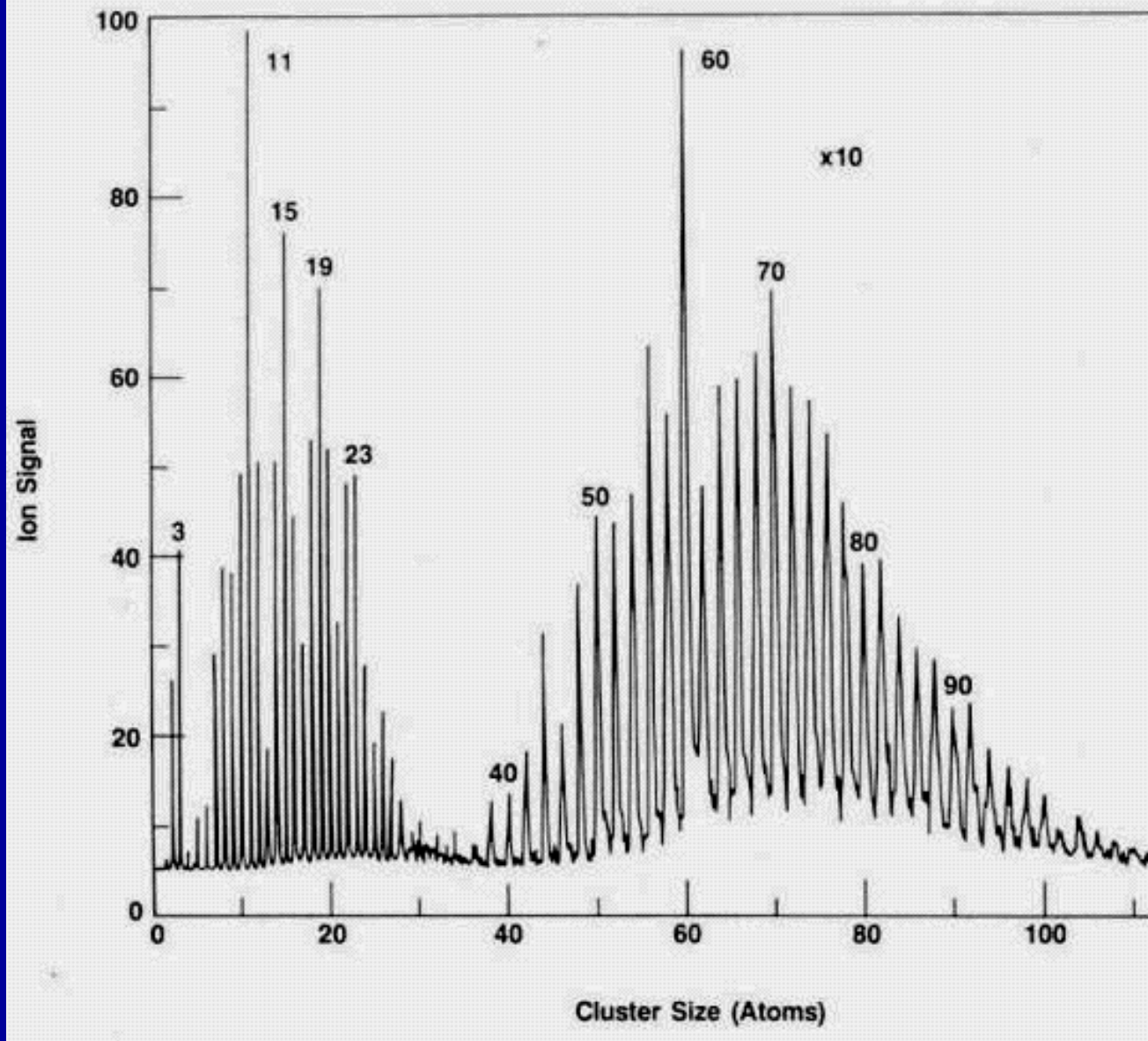
$$3V = 5p + 6h$$

$$6(F - E + V) = p$$

$$p = 12$$

$C_{20}, C_{24}, C_{26}, C_{28}, \dots, C_{60}, C_{70}, C_{2(10+h)} \dots$

$$V = 20 + 2h = 2(10 + h)$$



clusters with *magic* even-numbers of carbon atoms $36 < n < 150$ – fullerenes !!

E.A. Rohlfing, D.M. Cox, A. Kaldor, J. Chem. Phys. 81, 3322 (1984).

Euler's theorem for convex polyhedra

$$F - E + V = 2$$

where F - faces, E - edges, V -vertices

Fullerenes are closed-cage molecules of pure carbon in the shape of polyhedra with only pentagonal and hexagonal faces

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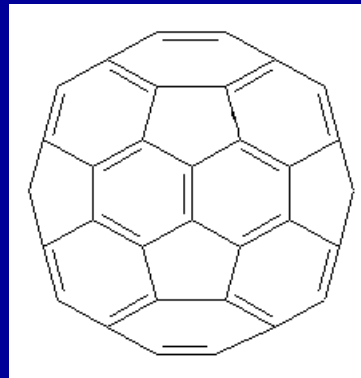
$$3V = 5p + 6h$$

$$6(F - E + V) = p$$

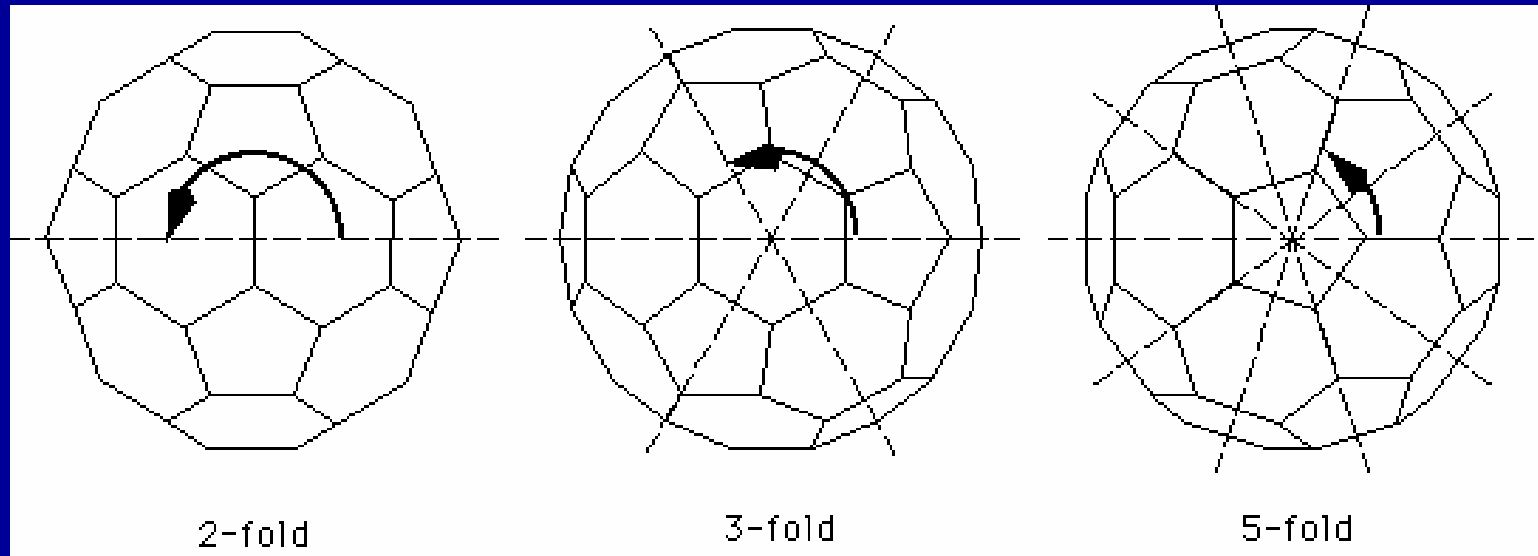
$$p = 12$$

$$V = 20 + 2h = 2(10 + h)$$

$$C_{20}, C_{24}, C_{26}, C_{28}, \dots, C_{60}, C_{70}, C_{2(10+h)} \dots$$



Isolated
pentagon rule



15 rotation axes

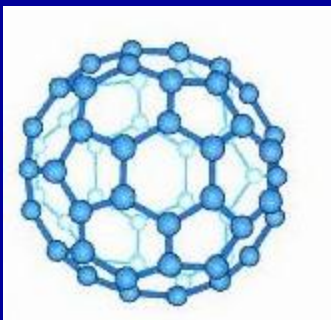
180°

10 rotation axes

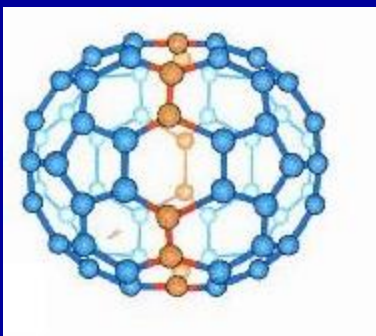
120°

6 rotation axes

$360/5=72^\circ$

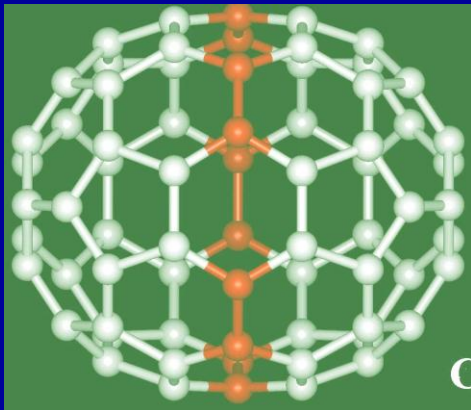
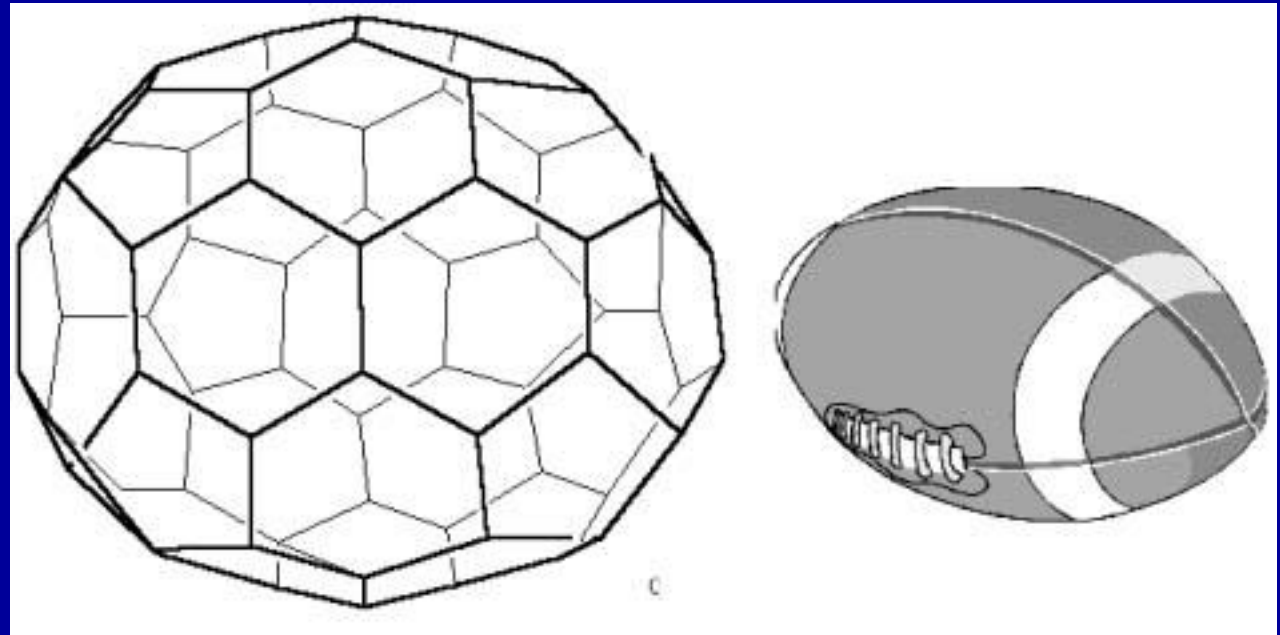


C_{60}

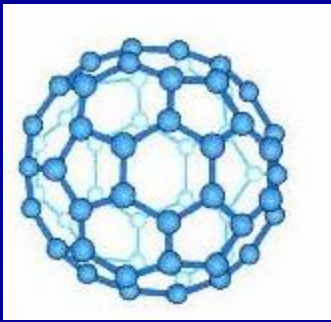


C_{70}

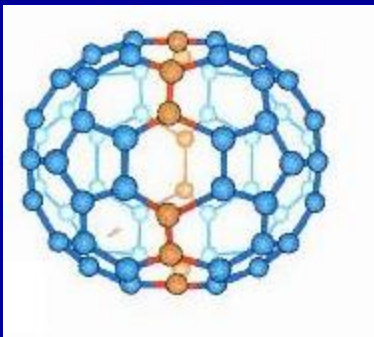
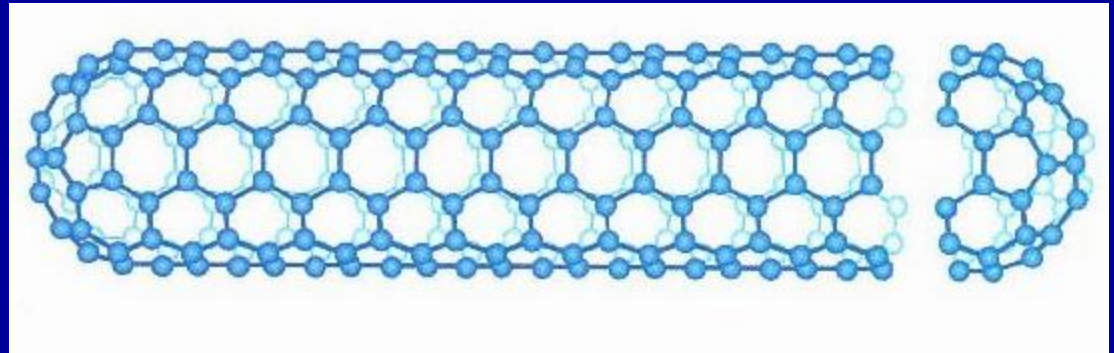
C_{70}



$C_{1000000}$ - carbon nanotube

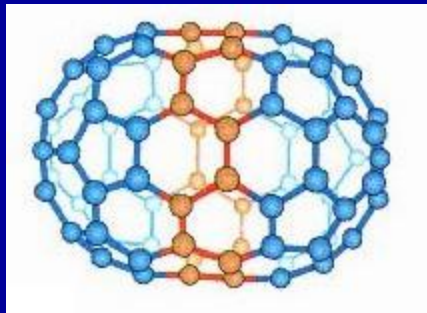


C_{60}

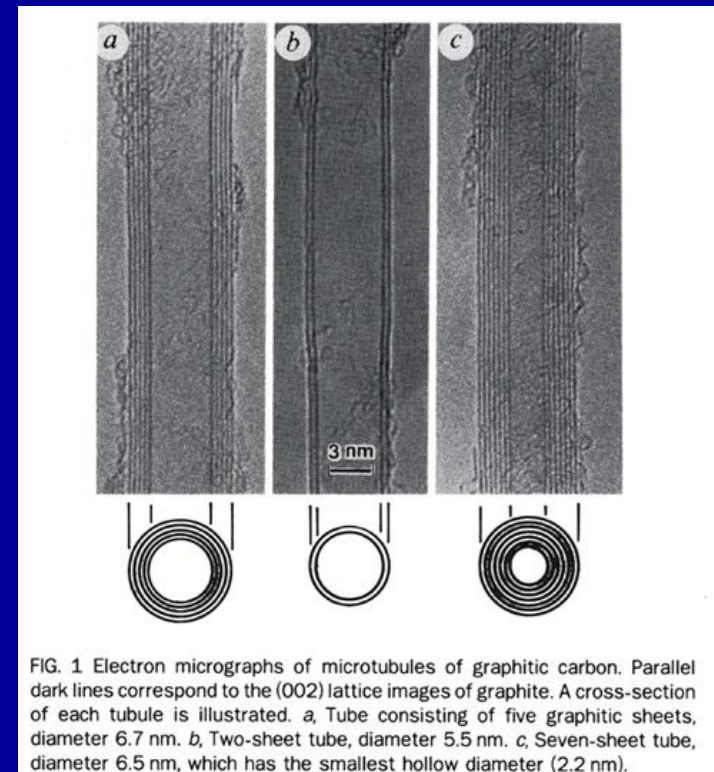
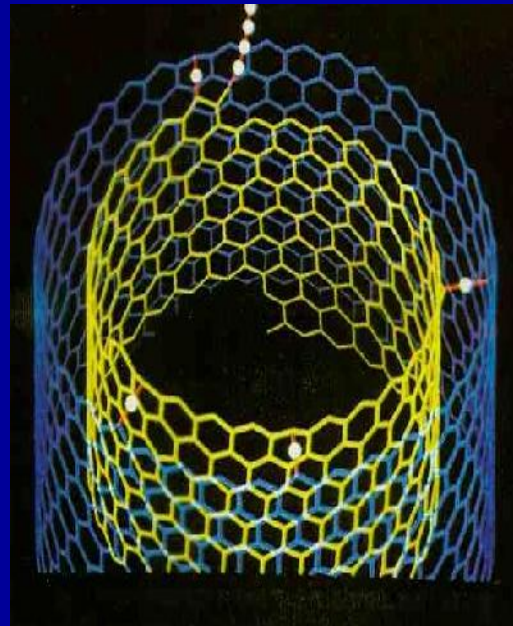


C_{70}

MWCN



C_{80}

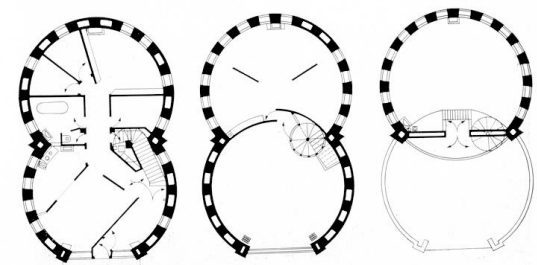
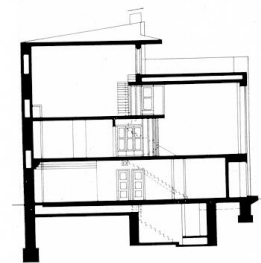
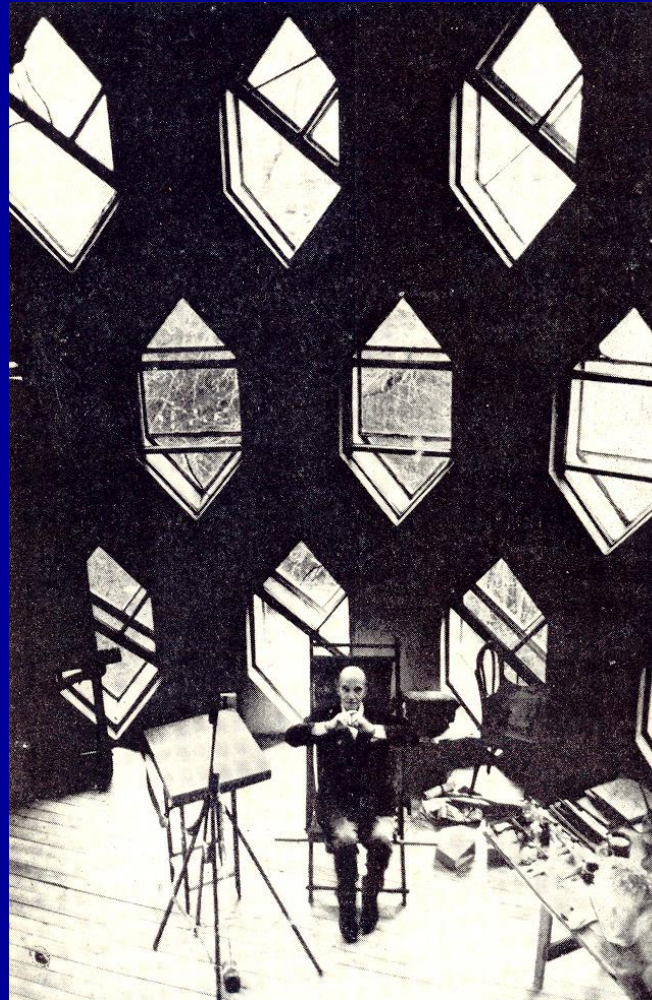
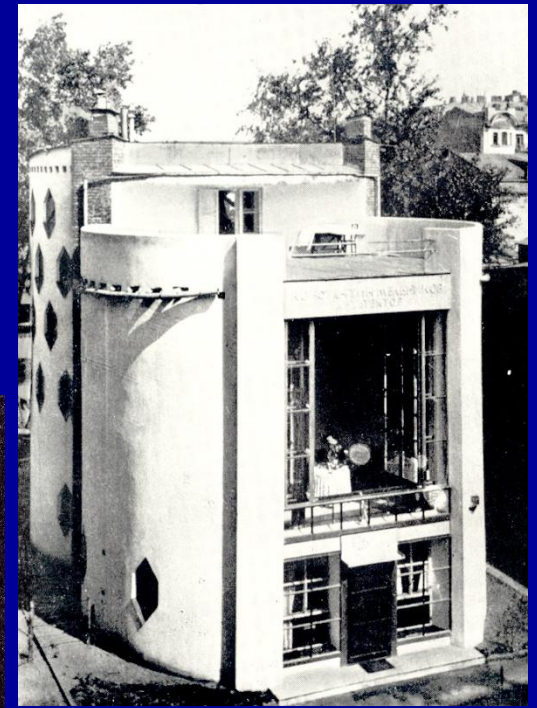
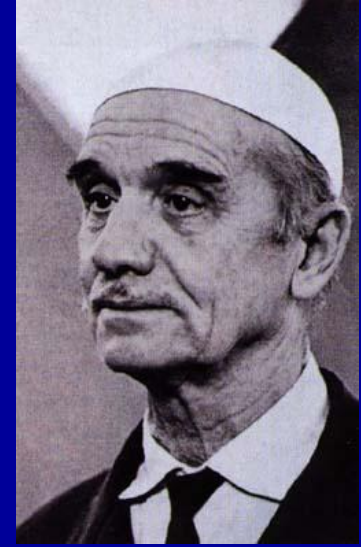


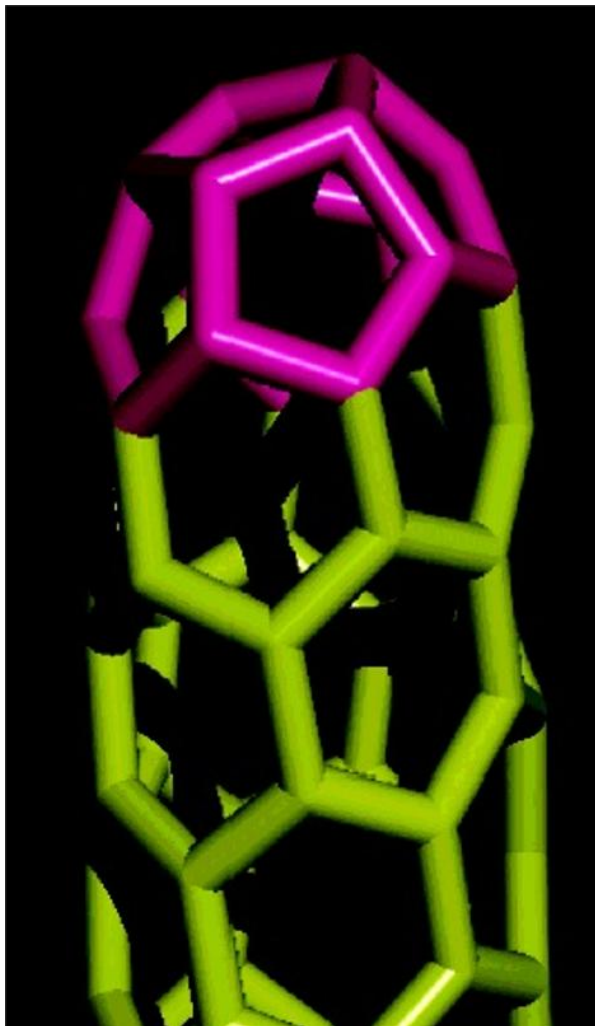
S. Iijima, Nature 354 (1991) 56

Konstantin Melnikov

1890-1974

1927





**CNT of smallest
diameter (0.43 nm)**

C₂₀

**T. Hayashi, Y. A. Kim, T. Matoba, M.
Esaka, K. Nishimura, T. Tsukada,
M. Endo and M. S. Dresselhaus.
Nanoletters, 3, 887 (2003).**



Liverpool



Florence

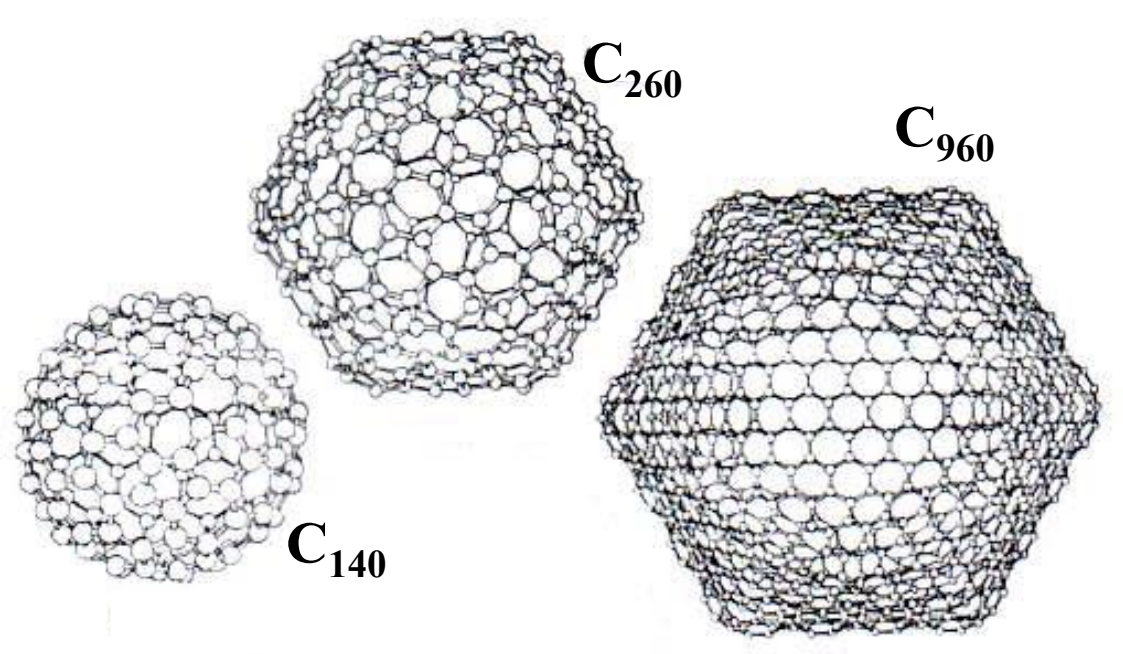


Beer-Sheva



Nati Ziv studio

Giant fullerenes with icosahedral symmetry



...One day I decided that we should build our own Buckminster Fuller domes, or rather molecular models of the giant fullerenes... Ken McKay... set about building C_{240} , C_{540} and later C_{960} and C_{1500} with icosahedral symmetry. When Ken came in with the model of C_{540} beautiful but I could not quite understand its shape – the model was not round like Buckminster Fuller's Montreal but had clear icosahedral tendencies. Indeed Ken's model had cusps focused at the 12 pentagons and from a distance had a definite polygonal outline... In fact as we looked more carefully at the infrastructure of Buckminster Fuller's domes we realized that the strut length in the vicinity of the pentagons had been adjusted to give them a smooth spheroidal shape.

H.W. Kroto, Nobel Lecture, December 7, 1996



Carbon onions

S. Iijima, J. Cryst. Growth, 50, 675 (1980).

Mail gilded guardian lions at the Forbidden City step their right front paws on an embroidered ball carved with a pattern of a fullerene (left) and a fullerene dual (right). Beijing, China, 1406-1420. Photo by E. Katz.

**E. A. Katz and B.-Y. Jin.
*Mathematical
Intelligencer*, 38, 61
(2016).**



fullerene



**fullerene
dual**

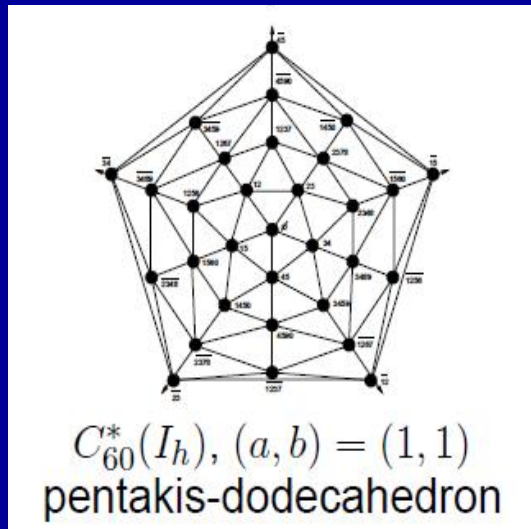


fullerene



fullerene dual – deltahedron
(with 6- and 5-valent vertices)

Icosadeltahedron (C_{20T}^*): dual of an icosahedral fullerene



Geodesic domes, viruses

Icosadeltahedra (dual fullerenes) in architecture



Fullerene	Geodesic dome
$F_{20}^*(I_h)$	One of Salvador Dali houses
$C_{60}^*(I_h)$	Artic Institute, Baffin Island
$C_{180}^*(I_h)$	Bachelor officers quarters, US Air Force, Korea
$C_{240}^*(I_h)$	U.S.S. Leyte
$C_{320}^*(I_h)$	Geodesic Sphere, Mt Washington, New Hampshire
$C_{500}^*(I_h)$	US pavilion, Kabul Afghanistan
$C_{720}^*(I_h)$	Radome, Artic dEW
$C_{3840}^*(I_h)$	Lawrence, Long Island
$C_{5120}^*(I_h)$	US pavilion, Expo 67, Montreal
$C_{6480}^*(I_h)$	Géode du Musée des Sciences, La Villete, Paris
$C_{6480}^*(I_h)$	Union Tank Car, Baton Rouge, Louisiana

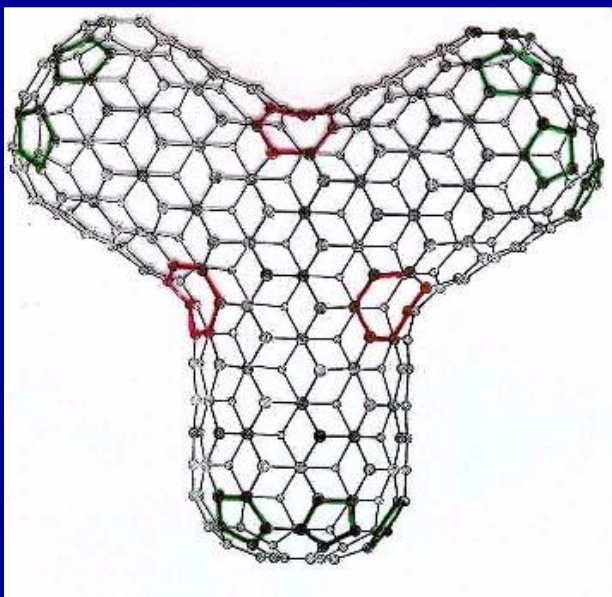
Porta Marina Roman Baths, Ostia, first half of the II century



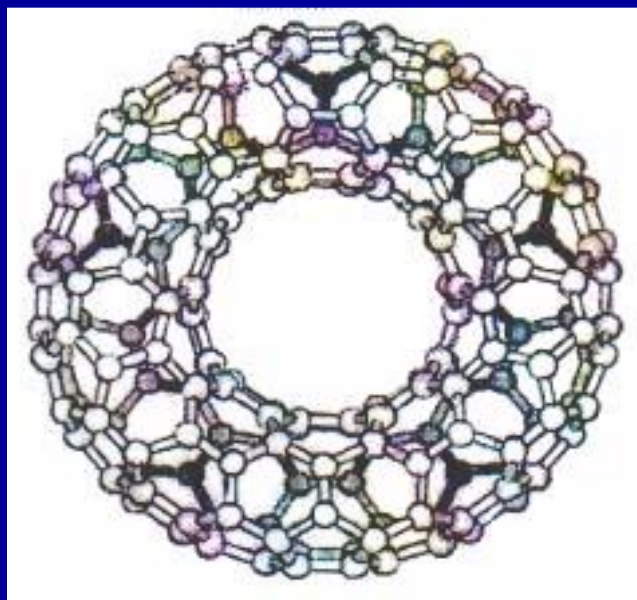
Photo by E. Katz

Fullerene-like structures with a negative curvature

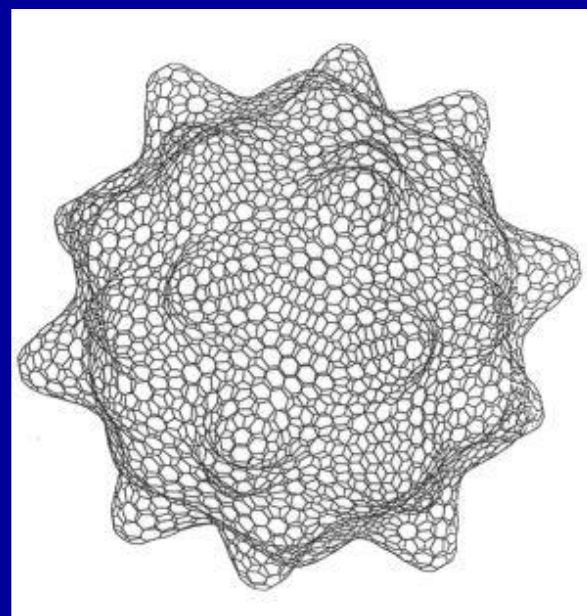
Pentagons, heptagons, hexagons



Y-nanotube

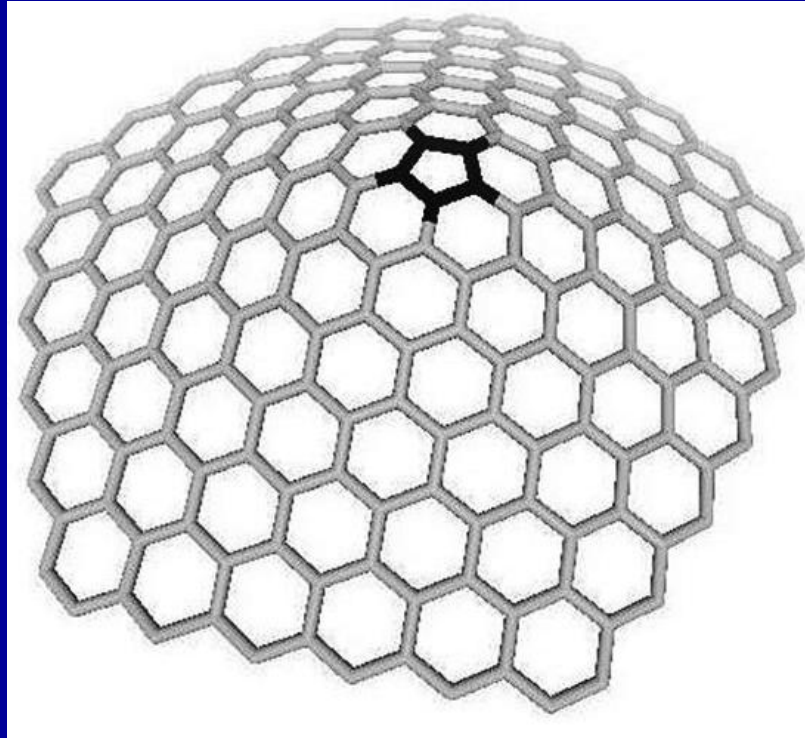


Carbon nano-torus

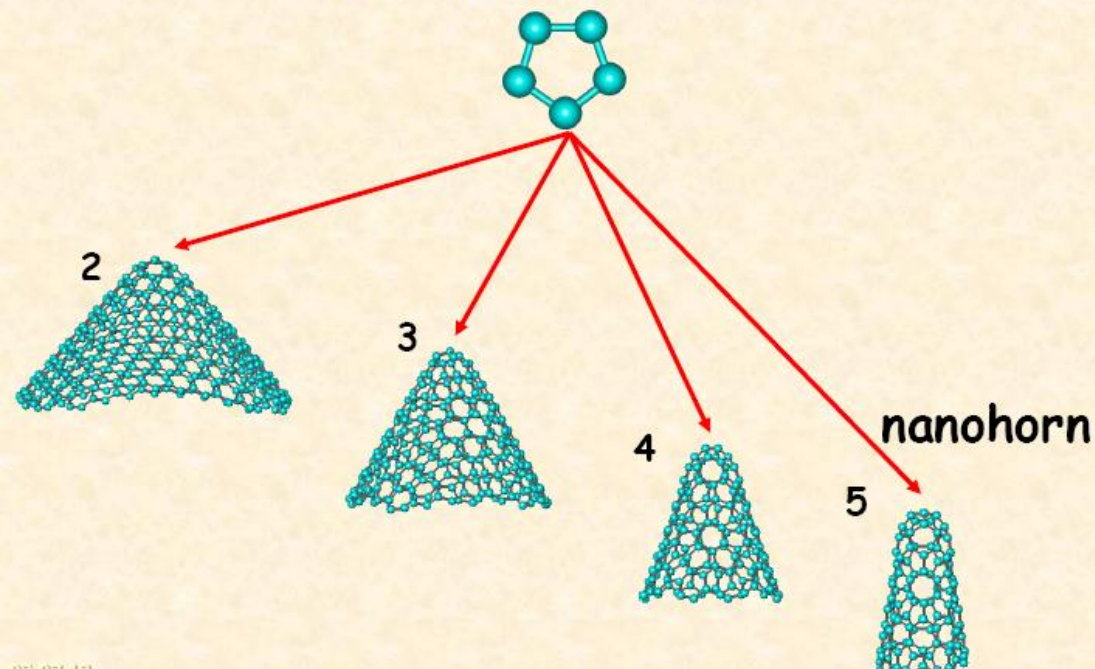


Star-like cluster

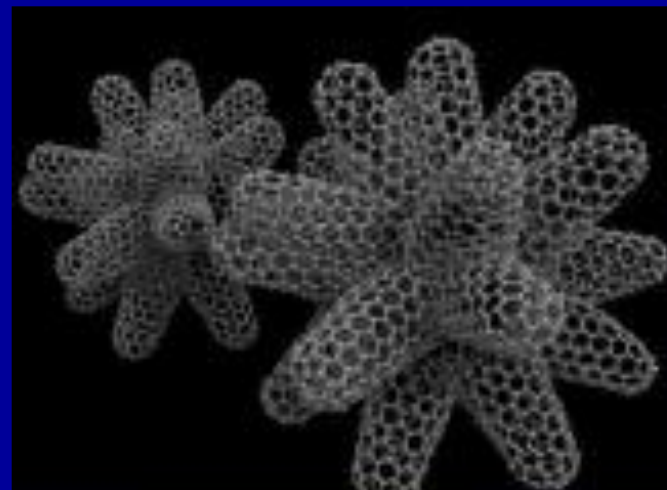
Carbon nano-cones



INFLUENCE OF NUMBER OF PENTAGONS



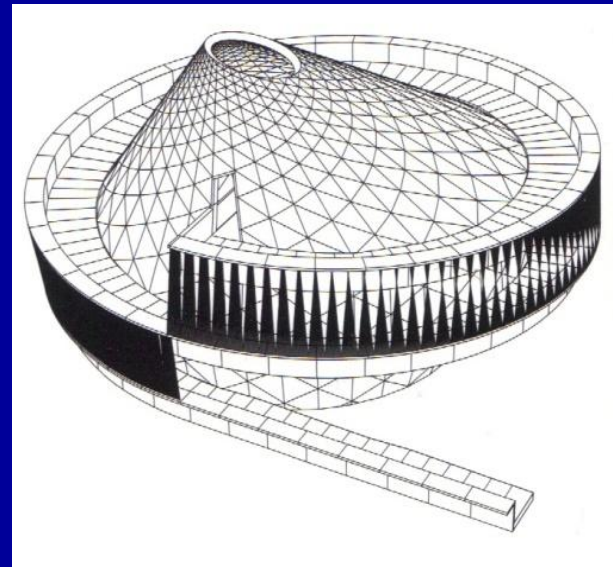
~100 nm



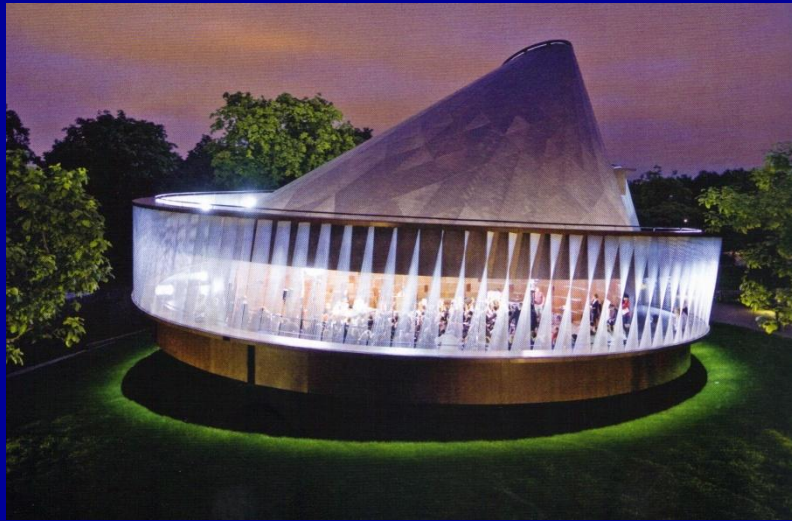
Kasuya D. et al. Selective production of single-wall carbon nanohorn aggregates and their formation mechanism. J. Phys. Chem. B, 2002, 106, 4947.



a



b



c



d

Architectural examples of conical geodesic-like structures. (a) Building of BMW Welt, Munich, Germany. Architect: Paul Kath, 2001-2007. (b-d) Serpentine Gallery pavilion, Kensington Gardens, London. Architects: Olafur Eliasson and Kjetil Thorsen, 2001-2007.

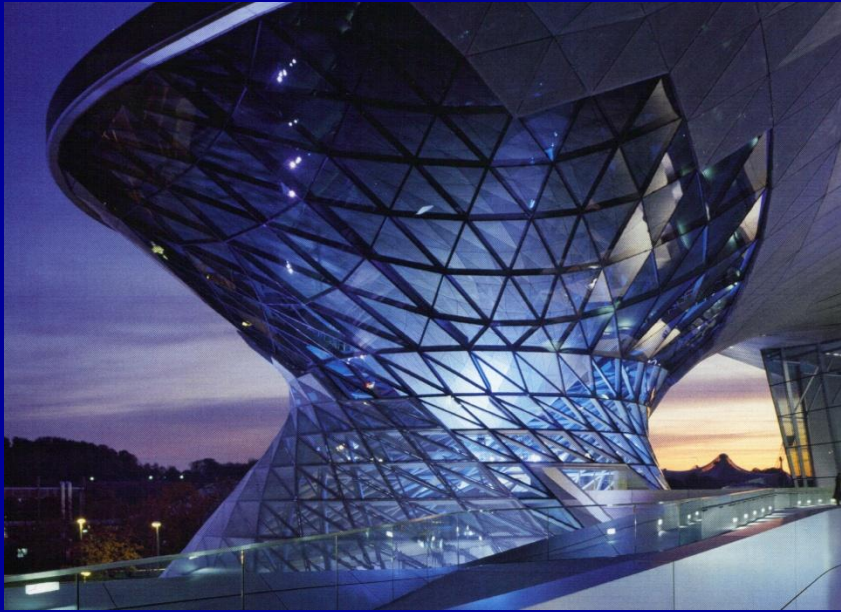
Photos from [Architecture now! 6. TASCHEN, 2009]

Architectural geodesic-like structures with negative curvature (1)



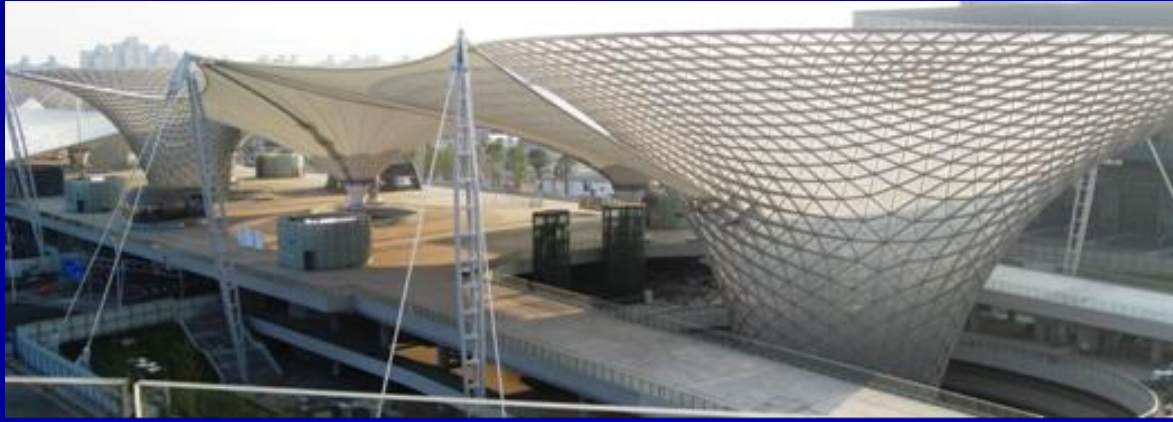
**Supermarket MyZeil, Frankfurt.
Architect: Massimiliano Fuksas
(2009). Photo by M. Sazonov. With
permission by M. Sazonov**

Architectural geodesic-like structures with negative curvature (2)



Building of BMW Welt, Munich, Germany. Architect: Paul Kath, 2001-2007. Photo from [Architecture now! 6. TASCHEN, 2009]

Architectural geodesic-like structures with negative curvature (3)



**“Expo-axis” at the Expo
2010, Shanghai,
China.**

Photo by E.Katz

**"Expo Axis" was the
main building at the
Expo 2010 in
Shanghai built by SBA
(architects) and
Knippers Helbig
(structural engineers).**



Photo from [http://en.wikipedia.org/wiki/Expo_2010]

Architectural geodesic-like structures with negative curvature (4)



**Museo Soumaya,
Mexico City,
architect Fernando Romero**

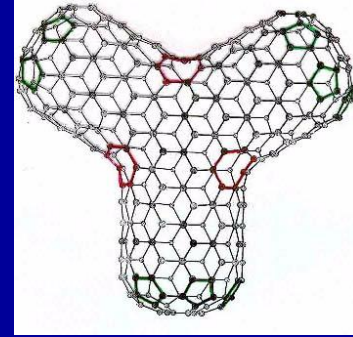
Pillow-like structures



**Eindhoven, The
Netherlands.
Photo by E. Katz**



Y-nanotube

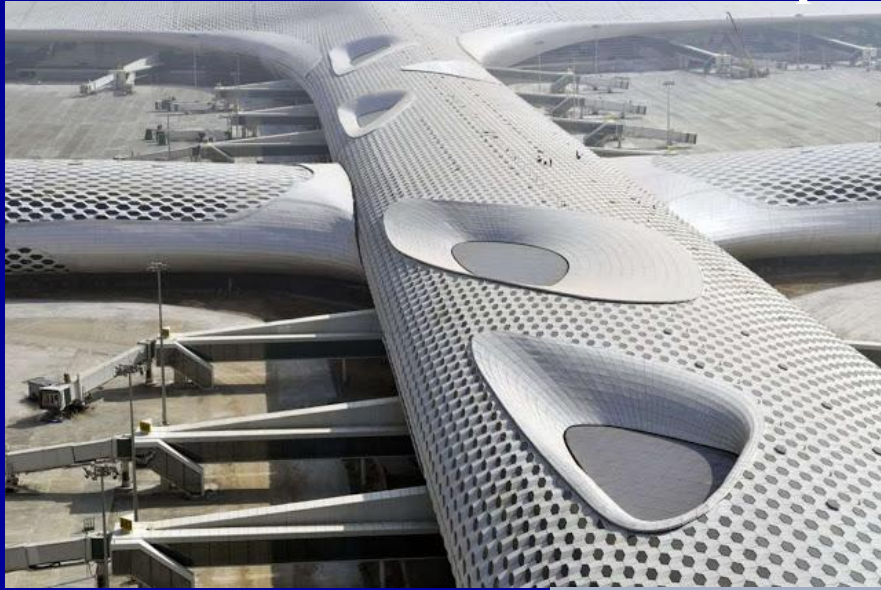


**Project:
URBAN ALLOY TOWERS,
NY (AMLGM, 2014)**

Residential typology rooted in the remnant spaces surrounding the intersection of transportation infrastructure, such as elevated train lines and freeway interchanges. In this space urban citizens can work, play, eat and rest within a pedestrian zone.



Terminal 3, Shenzhen International Airport, China (FUKSAS, 2013)
The sculptural 500,000 sq.m. terminal with 63 contact gates (1.5 km long, with roof spans of up to 80m).



Honeycomb shaped metal and glass panels allow natural light to filter through

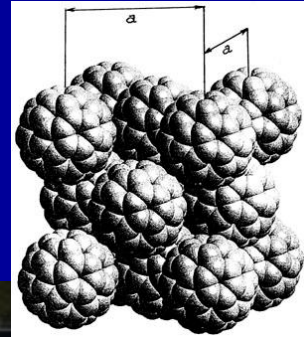


AAMI Park, Melbourne, Australia (Architect: Cox Architecture, 2010)

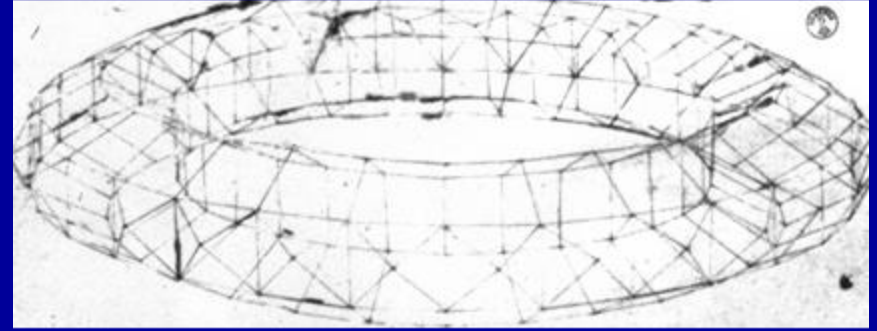
Multi-code sport campus (facility for 4 separate football stadiums, for example)



Fullerite



Polyhedral tori (mazzocchio) by Paolo Uccello (1397-1475)

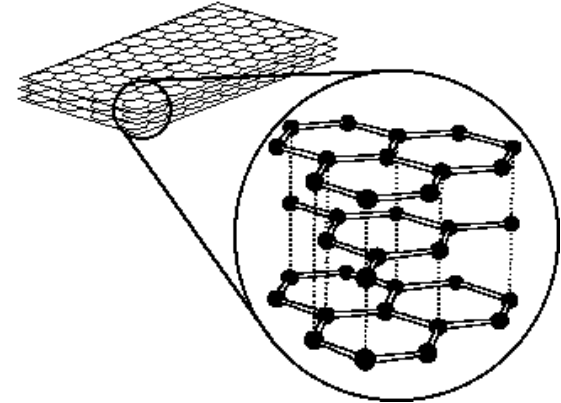


**Ubriachi chapel,
Santa Maria Novella,
Florence,
Photo by E. Katz**

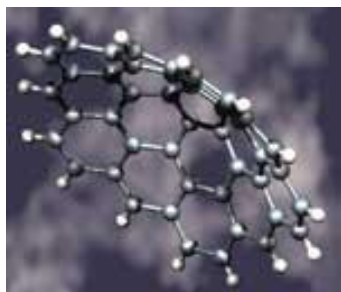


**Is it possible to build fullerenes from
other elements?**

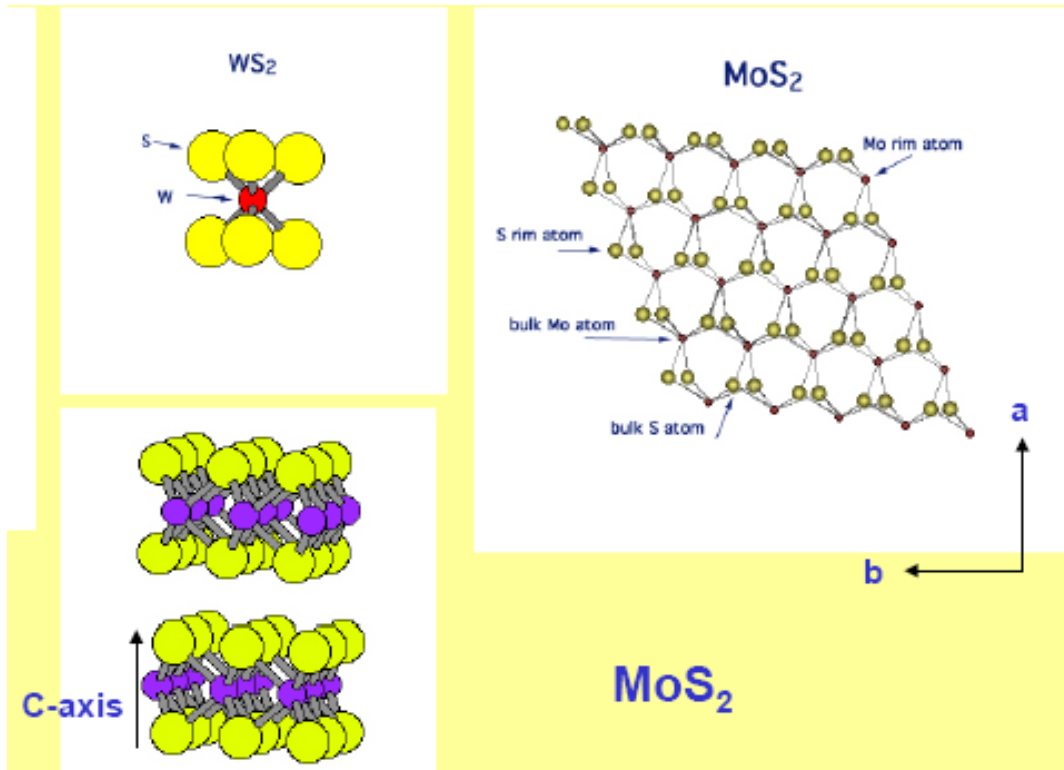
Non-carbon (inorganic) fullerenes?



Inorganic nanotubes and fullerene-like particles from 2-D layered compounds



Graphite

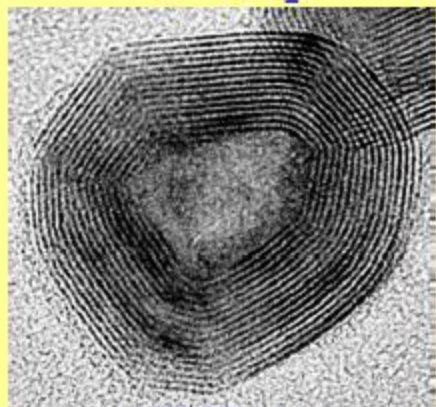


R.Tenne and co-workers, *Nature*, 360, 444 (1992); 365, 113 (1993); *J. Am. Chem. Soc.*, 116, 1914 (1994); *Science*, 267, 222 (1995)

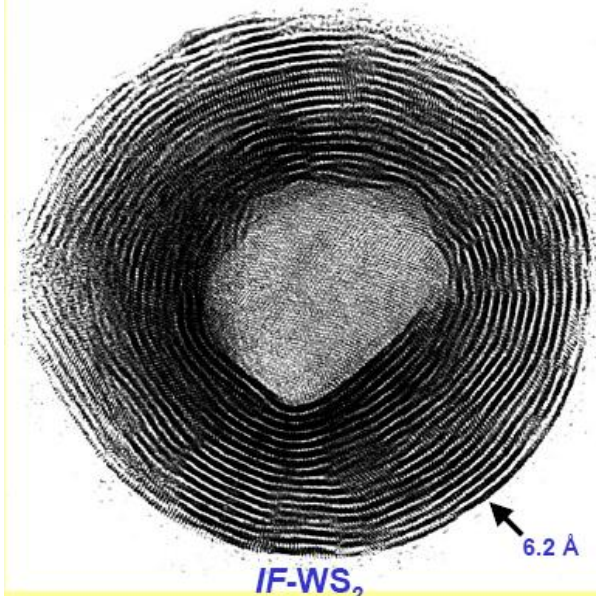
L. Margulis, M. Genut, G. Hodes, G. Salitra, M. Talianker, L. Gheber, M. Hershfinkel, J.L. Hutchison, V. Volterra, Y. Feldman, E. Wasserman, D.J. Srolovitz



IF-WS₂

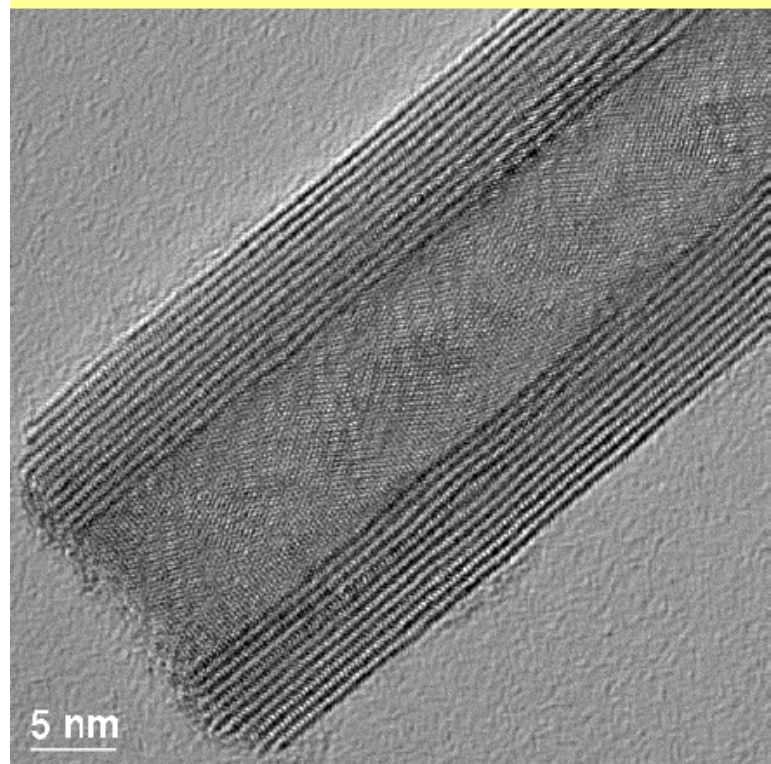


IF-MoS₂



IF-WS₂

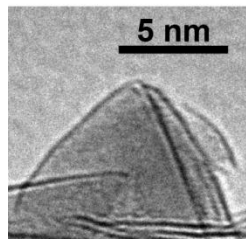
Multiwall WS₂ nanotube



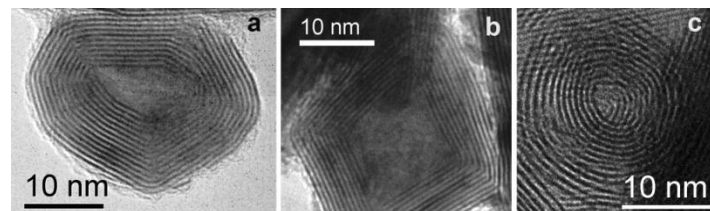
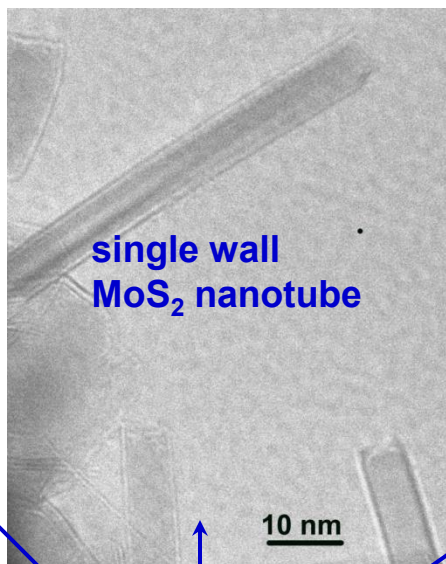
5 nm

Inorganic (non-carbon fullerenes)

MoS₂ fullerene-like nanoparticle



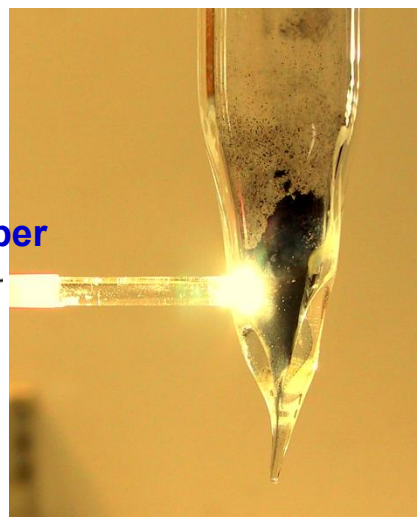
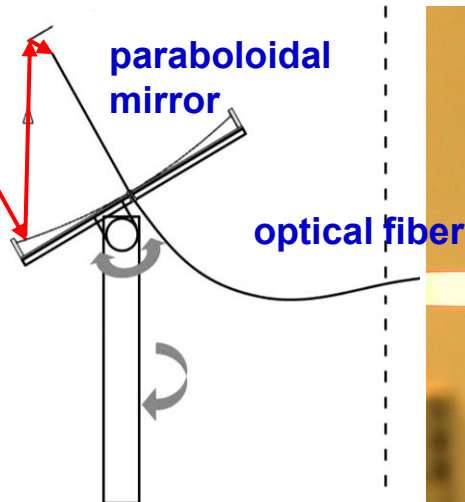
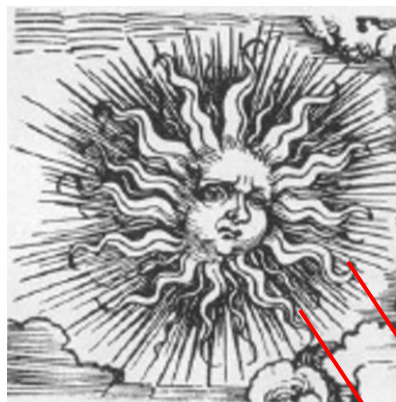
single wall MoS₂ nanotube



Nested Cs₂O fullerenes

Albu-Yaron, T. Arad, M. Levy, R. Popovitz-Biro, R. Tenne, J. M. Gordon, D. Feuermann, E. A. Katz, M. Jansen and C. Mühle. *Advanced Materials*, 18, 2993 (2006).

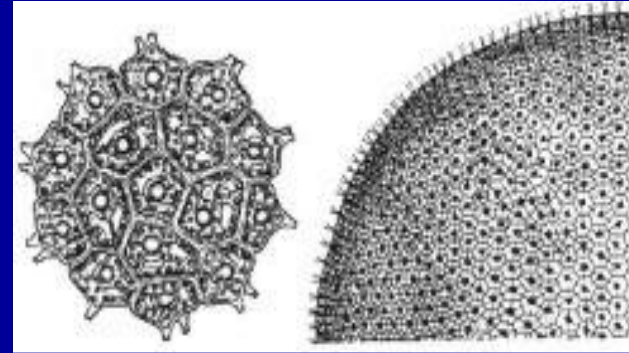
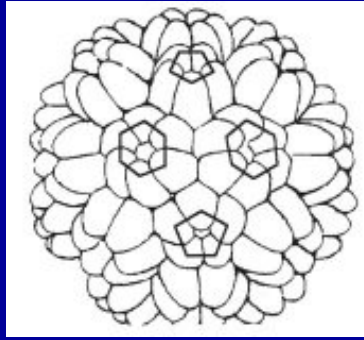
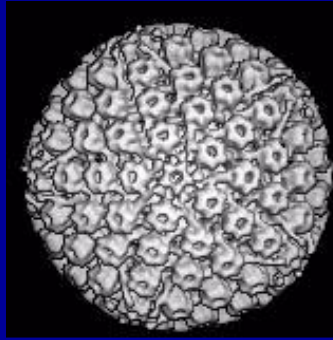
Solar assisted synthesis



J. M. Gordon, E. A. Katz, D. Feuermann, A. Albu-Yaron, M. Levy, R. Tenne. *Journal of Materials Chemistry*, 18, 458 (2008).

Fullerene-like structures in animate nature

Fullerene-like viruses



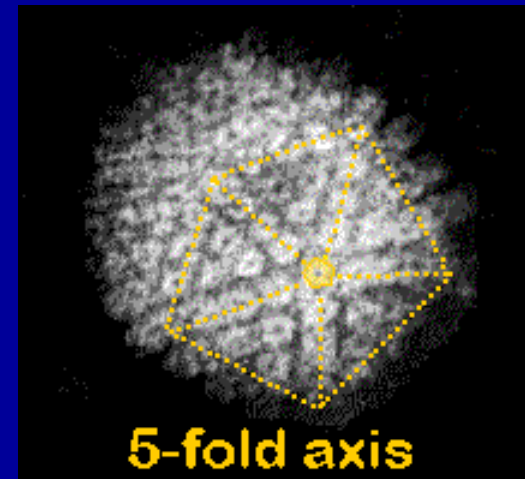
20-300 nm

Herpes

Could viruses be considered
as living organisms?

Yes: they have genetic material
(DNA, RNA) and can reproduce
themselves

No: they have no cell structure

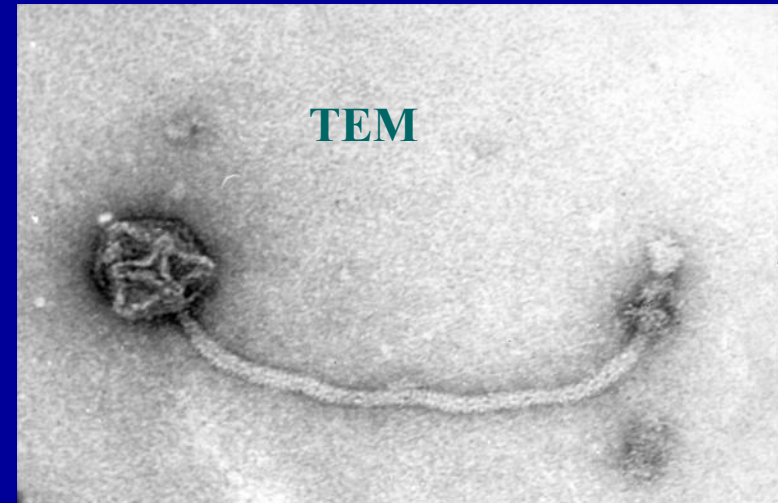
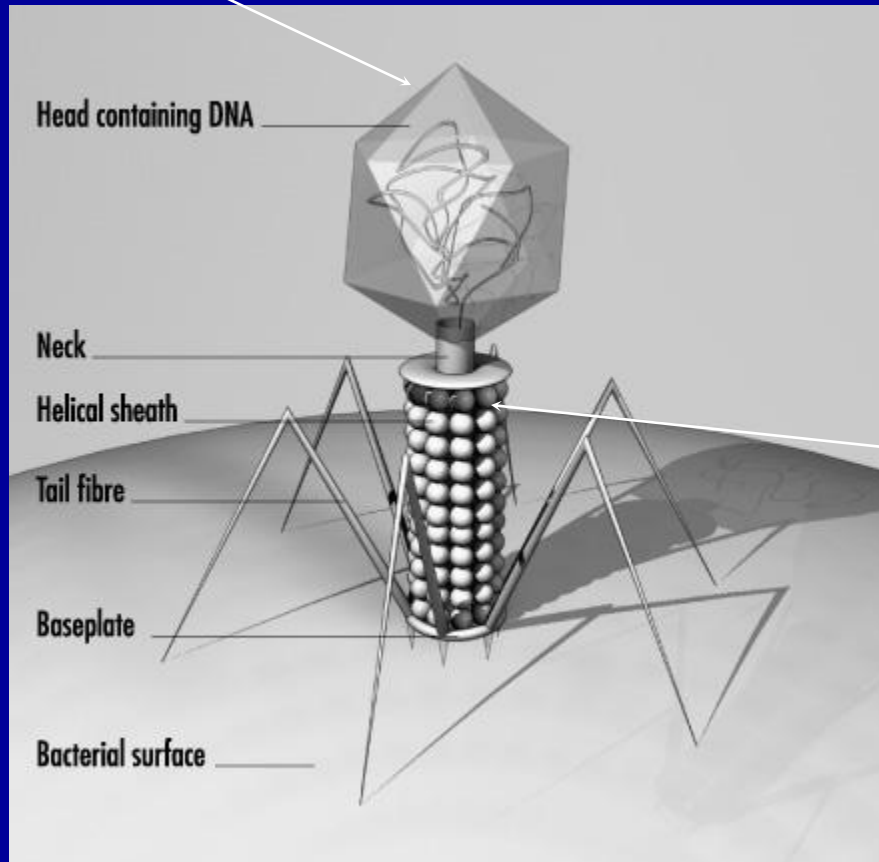


**Viruses are on the very boundary between
animate and abiotic nature.**

Bacteriophages – or phages - from 'bacteria' and Greek phagein, 'to eat') - viruses that infect bacteria.

Discovered by Félix d'Hérelle, 1917

40-140 nm



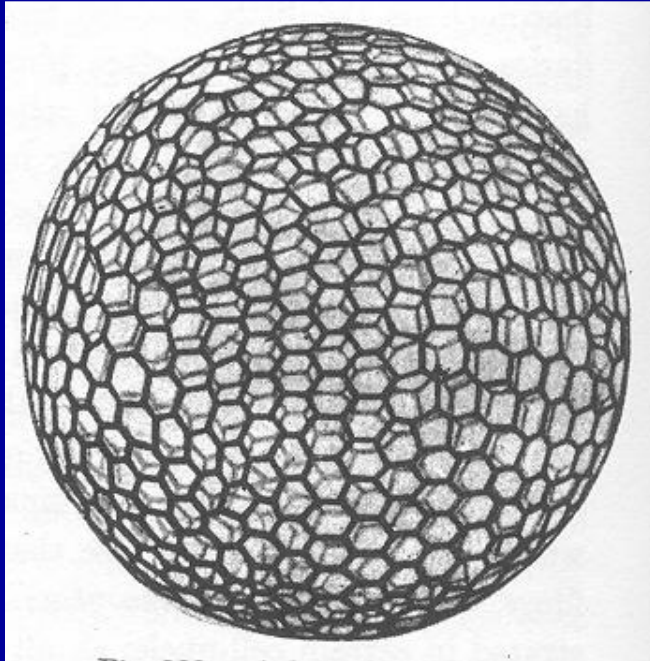
Nanotubes (10-40 nm diameter, 100-200 nm length)

Phage inject its DNA to a bacterial cell and suppress its DNA

Phage therapy

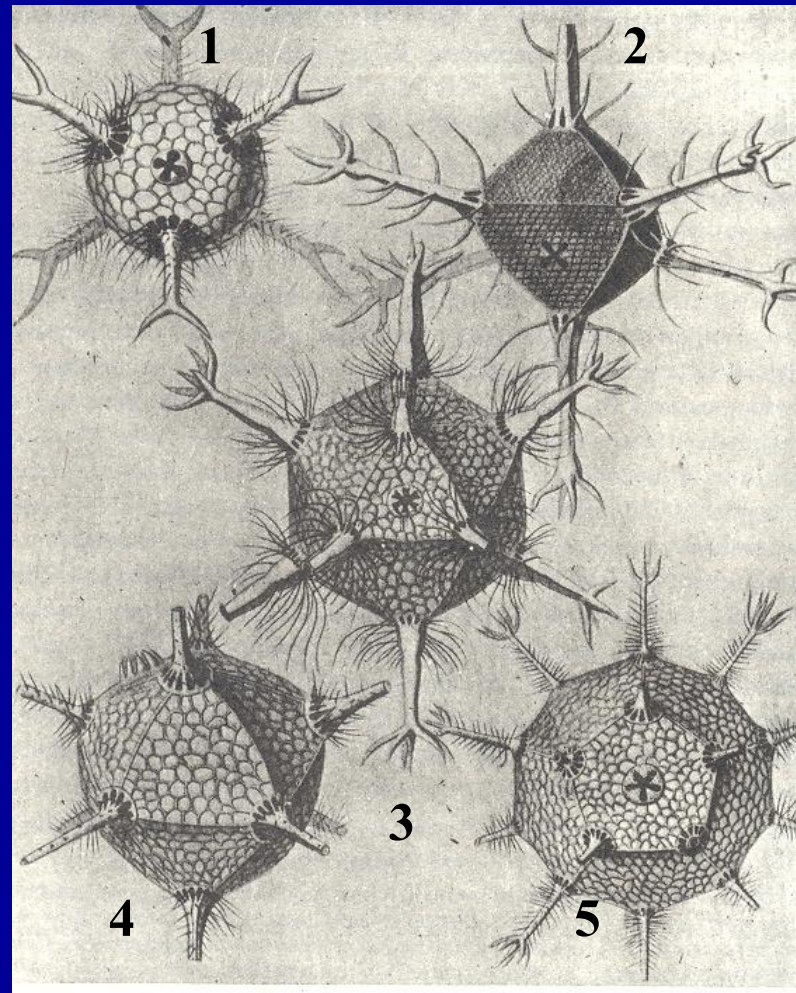
Radiolaria

40 μ m – 1mm



Skeleton of *Aulonia hexagona* Hkl.

From: D'arcy W. Thomson
“On Growth and Form”,
Cambridge Univ. Press,
1917



Skeletons of various radiolarians: 1. *Cicroporus sexfurcus*, 2. *C. octahedrus*, 3. *Circogonia icosahedra*, 4. *Cicrospathis novena*, 5. *Cicrorrhegma dodecahedra*.

From: E. Haeckel “Monograph of the Challenger Radiolaria”, 1987.

$$F - E + V = 2$$

$$p = 12$$

$$V = 20 + 2h = 2(10 + h)$$

Carbon clusters:

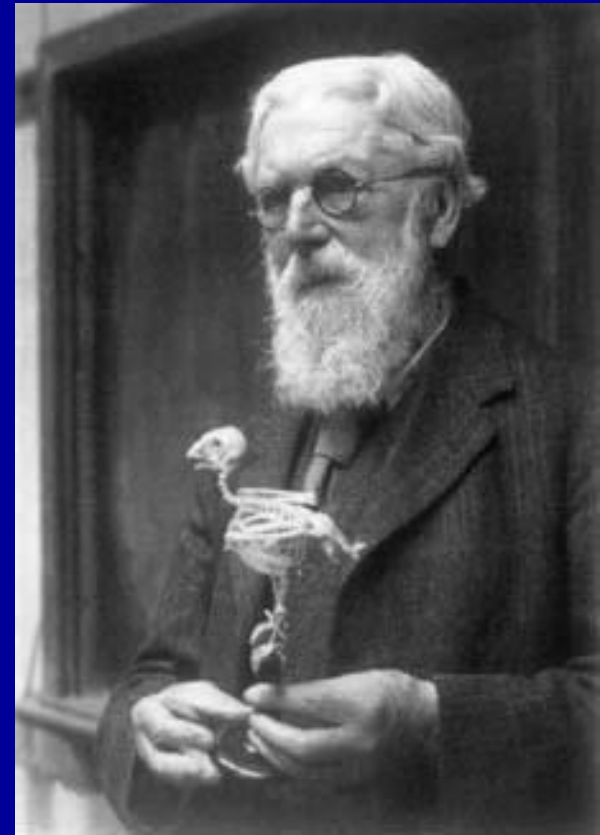
D.E.H. Jones, 1966 → S. Iijima, 1980 → H. Kroto, 1985-1992

Radiolaria:

**D'arcy W. Thomson “On Growth and Form”, Cambridge Univ. Press, 1917;
2nd ed., 1942; posthumous 3rd ed., 1952;
4th ed., 1959; 5th ed., 1963, 6th ed., 1992.**

Scientific relay race:

D'arcy Thomson → Jones → Iijima → Kroto



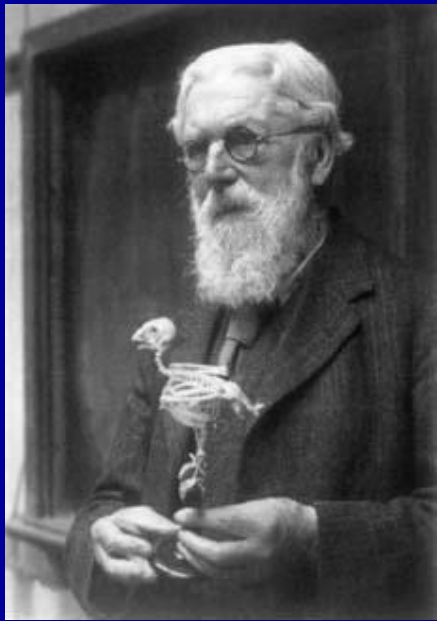
Dmitry D. Morduhai-Boltovskoi (1876–1952):



Prominent mathematician,
historian of mathematics,
philosopher
(with a strong background in
classics);
the author of the 1st complete,
definitive translation of
Euclid's "*Elements*"
into Russian (1948-50)

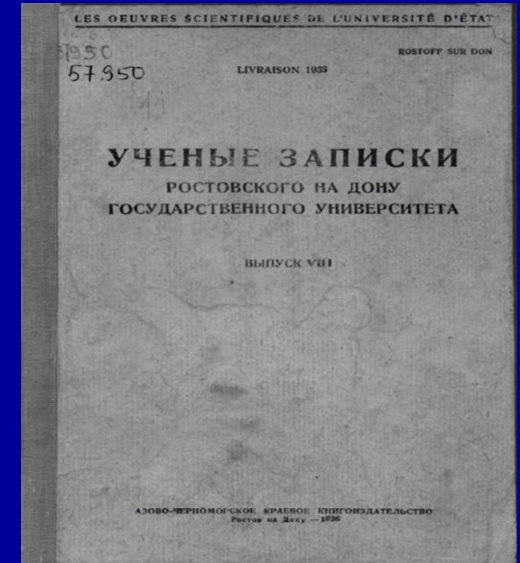
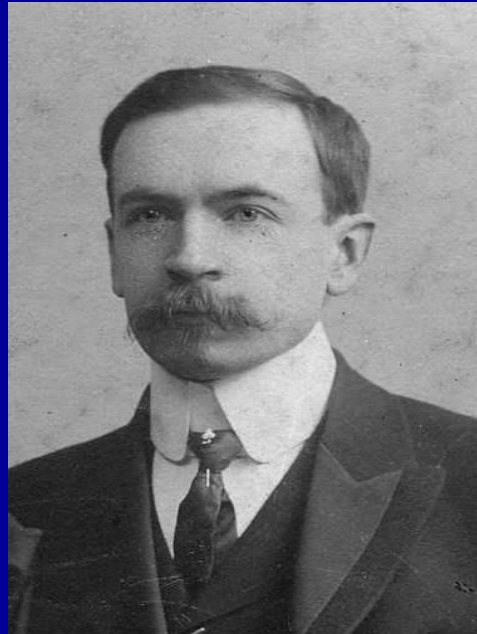
1906

<http://pyrkovve.narod.ru/index.html>



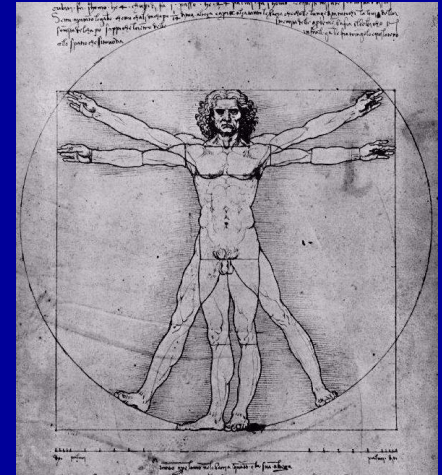
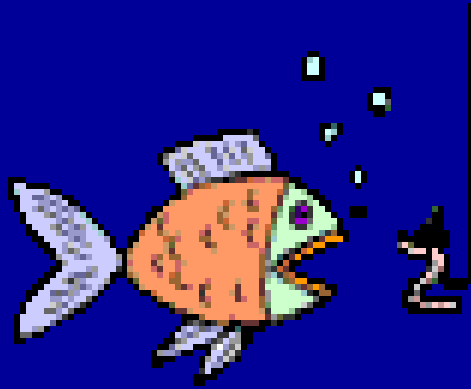
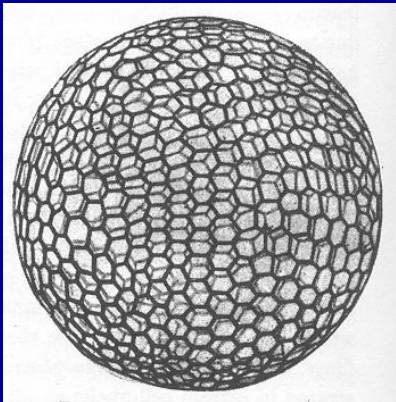
D'arcy W. Thomson
“On Growth and Form”,
Cambridge Univ. Press, 1917;
2nd ed., 1942;

D. D. Morduhai-
Boltovskoi
“Geometry of
Radiolaria”, 1936



E. A. Katz. *Mathematical Intelligencer*, 36, 34 (2014).

Evolution of living organisms



Decrease of symmetry

Thanks a lot for your attention!